Tôhoku Math. Journ.
26 (1974), 461-482.

EQUIVARIANT CHARACTERISTIC NUMBERS AND INTEGRALITY THEOREM FOR UNITARY T^n-MANIFOLDS

AKIO HATTORI

(Received July 7, 1973)

1. Introduction. Let G be a compact Lie group and M a compact unitary (i.e., weakly complex) G-manifold. Thus G acts on M by diffeomorphisms preserving the given complex structure of the stable tangent bundle of M. The stable tangent bundle, with this G-action, defines an element $\tilde{\tau}M$ in $\tilde{K}_G(M)$ where $\tilde{K}_G(M)$ denotes the kernel of the augmentation $K_G(M) \xrightarrow{\dim} H_0(X, \boldsymbol{Z})$. If $\boldsymbol{t} = (t_1, t_2, \cdots)$ is a sequence of indeterminates and V is a complex G-vector bundle V over M, we define $\gamma_t(V - \dim V)$ in $K_G(M)[[\boldsymbol{t}]]$ by

$$\gamma_t(V - \dim V) = \prod_{i=1}^{\dim V}(1 + t_1(V_j - 1) + t_2(V_i - 1)^2 + \cdots)$$

where V is written formally as

$$V = \sum_{i=1}^{\dim V} V_i \ .$$

γ_t extends to a map

$$\gamma_t \colon \tilde{K}_G(M) \to K_G(M)[[\boldsymbol{t}]]$$

such that

$$\gamma_t(x + y) = \gamma_t(x)\gamma_t(y) \ .$$

Suppose that M is closed (i.e., compact and without boundary) and let $p_{M!} \colon K_G(M) \to K_G^* = K_G^*(\text{point})$ be the Gysin homomorphism of $p_M \colon M \to$ point. The element $p_{M!}(\gamma_t(\tilde{\tau}M))$ in $K_G^*[[\boldsymbol{t}]]$ turns out to be an invariant of the G-equivariant bordism class the of unitary G-manifold M so that the assignment $[M] \mapsto p_!(\gamma_t(\tilde{\tau}M))$ defines a homomorphism

$$\rho \colon U_*^G \to K_G^*[[\boldsymbol{t}]] \ ,$$

where U_*^G is the bordism ring of closed unitary G-manifolds. The homomorphism ρ also preserves the ring structure. The coefficients of the formal power series $\rho[M]$ are called equivariant K-theory characteristic numbers of $[M] \in U_*^G$. Note that the coefficient ring K_G^* has trivial odd-dimensional component K_G^{-1} and $K_G^* = K_G$ is canonically isomorphic to