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THE UNSTABLE DIFFERENCE BETWEEN HOMOLOGY COBORDISM AND PIECEWISE LINEAR BLOCK BUNDLES

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0. Introduction and statement of results. N. Martin and C. R. F. Maunder [9] developed the theory of homology cobordism bundles which is an adequate bundle theory in the category of polyhedral homology manifolds. They introduced certain Δ-sets $H(n)$ which play the role of "structure groups" in the bundle theory. A typical k-simplex of $H(n)$ is a homology cobordism bundle-automorphism of the product bundle $\Delta^k \times S^{n-1}$, or equivalently, a homology cobordism bundle over $\Delta^k \times I$ which is the product bundle over $\Delta^k \times \{0, 1\}$. According to N. Martin [10], the structure groups $\widetilde{PL}(n)$ of PL n-block bundles are homotopically equivalent to sub-Δ-sets $\overline{PL}(n)$ of $H(n)$. By definition a typical k-simplex of $\overline{PL}(n)$ is a PL n-block bundle over $\Delta^k \times I$ which is the product bundle over $\Delta^k \times \{0, 1\}$.

Our main result is the following

THEOREM 1. *If* $n \geqq 3$, *we have*

$$\pi_k(H(n), \overline{PL}(n)) = \begin{cases} 0 & (k \neq 3) \\ \mathscr{H}^3 & (k = 3)\ , \end{cases}$$

where $\mathscr{H}^3$ *is the abelian group of PL H-cobordism classes of oriented PL homology* 3-*spheres.*

This improves the result of [10] in the unstable ranges. Theorem 1 will be proved in §1.

Now for the case $n = 2$, let $\mathscr{G}_k$ be the ordinary knot cobordism group of PL $(k, k+2)$-sphere pairs and let $\mathscr{G}_k^H$ be the knot cobordism group of PL homology $(k, k+2)$-sphere pairs; any element of $\mathscr{G}_k^H$ is represented by a locally flat pair (M^k, N^{k+2}) consisting of oriented PL homology k- and $(k+2)$-spheres. Such pairs (M_1^k, N_1^{k+2}) and (M_2^k, N_2^{k+2}) represent the same element of $\mathscr{G}_k^H$ if and only if the connected sum $(M_1^k \# -M_2^k, N_1^{k+2} \# -N_2^{k+2})$ bounds a locally flat pair of acyclic manifolds (V^{k+1}, W^{k+3}). Also $\mathscr{G}^{AH}$ denotes the subgroup of $\mathscr{G}_1^H$ whose element is represented by a pair $(M^1,$

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