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GLOBAL ANALYTIC-HYPOELLIPTICITY OF THE $\bar{\partial}$-NEUMANN PROBLEM

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Introduction. The (real-)analytic behavior (near the boundary) of solutions of the so-called $\bar{\partial}$-Neumann problem seems to have been unknown. In this paper we show that the global analytic-hypoellipticity (up to the boundary) holds on certain domains in $\boldsymbol{C}^n$ with analytic boundaries.

A systematic study of the $\bar{\partial}$-Neumann problem was made by Kohn [3], and the most difficult part of his work was the proof of the C^∞ hypoellipticity (up to the boundary). Soon after, Kohn and Nirenberg [5] gave an elegant proof of the C^∞ hypoellipticity by establishing the so-called subelliptic estimate. Their method is today used for various problems as the standard technique. However, it seems difficult, even if possible, to deduce the analytic-hypoellipticity of the $\bar{\partial}$-Neumann problem from the subelliptic estimate.

Under these circumstances we introduce in Lemma 2 a certain special vector field tangential along the boundary, which can be constructed in the case the Levi form is non-degenerate. It possesses the properties nice enough to carry out the commutator estimates (Lemmas 4 and 5), and these estimates together with the a priori estimate (Lemma 1) lead us in the usual way (see, e.g., Morrey and Nirenberg [6]) to our result. Our a priori estimate is suggested by a paper of Kohn [4].

It should be mentioned that the local problem still remains unsolved, and our method may not be applicable.

1. Statement of the theorem. Let $M \subset \boldsymbol{C}^n$ be a bounded domain whose boundary bM is regularly embedded in $\boldsymbol{C}^n$ with real codimension one. In all that follows we shall assume that the standard hermitian metric is given in $\boldsymbol{C}^n$ and that bM is analytic.

Let r denote the geodesic distance to bM measured as positive outside M and negative inside M, and normalized so that $|dr|^2 = 2$ near bM, where $|\cdot|$ is the length defined by the metric in $\boldsymbol{C}^n$. With a sufficiently small constant $\rho > 0$, we denote by Ω'_ρ the tubular neighborhood $bM \times (-\rho, \rho)$, i.e., $\{\mathrm{P} \in \boldsymbol{C}^n; -\rho < r(\mathrm{P}) < \rho\}$, and we set $\Omega_\rho = \bar{M} \cap \Omega'_\rho$, where $\bar{M}$