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A CONVOLUTION MEASURE ALGEBRA ON THE UNIT DISC

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1. Introduction. Let D be the unit disc $D = \{z = x + iy; x^2 + y^2 \leqq 1\}$ and m_α be the positive measure of total mass one on D defined by

$$dm_\alpha(z) = \frac{\alpha + 1}{\pi}(1 - x^2 - y^2)^\alpha dxdy ,$$

where α is a positive real number. Let $M(D)$ be the space of all bounded regular complex valued Borel measures on D. $M(D)$ is a Banach space with the total variation norm $\| \mu \| = \int_D d|\mu|(z)$ for $\mu \in M(D)$. Denote $L_\alpha^1 = L^1(D, m_\alpha)$. Then L_α^1 is identified with a subspace of $M(D)$ by the map $f \mapsto f dm_\alpha$ of L_α^1 to $M(D)$. The mapping is isometric, since $\| f \|_{L_\alpha^1} = \int_D |f(z)| dm_\alpha(x) = \| f dm_\alpha \|$.

For each point z in D, the operator T_z, called generalized translation, is defined by

$$(1) \qquad T_z f(\zeta) = \frac{\alpha}{\alpha + 1}\int_D f\left(\bar{z}\zeta + \sqrt{1 - |z|^2}\sqrt{1 - |\zeta|^2}\,\xi\right)\frac{dm_\alpha(\xi)}{1 - |\xi|^2} ,$$

for f in the space of all continuous functions $C(D)$. By a change of variable, if z and ζ are in the interior of D, we obtain

$$T_z f(\zeta) = \int_D f(\xi) E_\alpha(z, \zeta, \xi) dm_\alpha(\xi) ,$$

where

$$E_\alpha(z, \zeta, \xi) = \begin{cases} \dfrac{\alpha}{\alpha + 1}\dfrac{(1 - |z|^2 - |\zeta|^2 - |\xi|^2 + 2\Re(\bar{z}\zeta\bar{\xi}))^{\alpha - 1}}{(1 - |z|^2)^\alpha (1 - |\zeta|^2)^\alpha (1 - |\xi|^2)^\alpha} , \\ 0. \end{cases}$$

The first value is assigned only if ξ is in the disc of the center $\bar{z}\zeta$ and of radius $\sqrt{1 - |z|^2}\sqrt{1 - |\zeta|^2}$. By the definition,

$$(2) \qquad E_\alpha(z, \zeta, \xi) \geqq 0 , \quad z, \zeta \in \text{interior of } D, \xi \in D ,$$

$$(3) \qquad \int_D E_\alpha(z, \zeta, \xi) dm_\alpha(\xi) = 1 .$$