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ON PARABOLIC SUBGROUPS OF CHEVALLEY GROUPS OVER LOCAL RINGS

KAZUO SUZUKI

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Introduction. Let G be a Chevalley-Demazure group scheme associated with a connected complex semi-simple Lie group G_C (as for definition, see [1] 1.1), Δ be the root system associated with G and a maximal torus T of G, and R be a commutative ring with a unit. We shall fix a fundamental root system Π of Δ once for all. Denote by $x_\alpha(t)$ the unipotent element of $G(R)$ associated with a root α of Δ and $t \in R$. Let $V(R)$ be the subgroup of $G(R)$ generated by $x_\alpha(t)$ for all negative roots α of Δ and all $t \in R$. Then a subgroup P of $G(R)$ containing $V(R)T(R)$ is called a parabolic subgroup of $G(R)$ associated with Π. Following J. Tits, it is well known that if R is a field, then the set of parabolic subgroups of $G(R)$ associated with Π is lattice isomorphic to the family of subsets of Π.

N. S. Romanovskii [4] has given a discription of parabolic subgroups of $GL_n(R)$ for a local ring R. In this note, for a simple Chevalley-Demazure group scheme G and a local ring R, we shall give a generalization of the Tits' theorem in the same situation as Romanovskii's result. The main theorem is stated in Section 1, and we shall prove our main theorem in Sections 2 and 3. The author wishes to express his hearty thanks to professor E. Abe for his many helpful comments and encouragement.

1. The statement of the main theorem.

1.1. Let G be a Chevalley-Demazure group scheme and R be a commutative ring with a unit. A collection of ideals $\{\mathfrak{A}_\alpha\}_{\alpha \in \Delta}$ which corresponds bijectively to the set Δ of roots, is called a carpet of R associated with Δ. Furthermore, a carpet $\{\mathfrak{A}_\alpha\}_{\alpha \in \Delta}$ is called a permissible (resp. semi-permissible) carpet associated with (Δ, Π), if the following conditions (1) and (2) (resp. (1) and (2')) are satisfied,

(1) for any roots α and β of Δ such that $\alpha + \beta \in \Delta$

$$\mathfrak{A}_\alpha \mathfrak{A}_\beta \subset \mathfrak{A}_{\alpha+\beta}$$

(2) for each negative root α of Δ, $\mathfrak{A}_\alpha = R$,