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STABILITY AND EXISTENCE OF ALMOST PERIODIC SOLUTIONS OF SOME FUNCTIONAL DIFFERENTIAL EQUATIONS

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1. Introduction. Assuming the uniqueness of solutions for the initial value problem, several authors discussed that the existence of a bounded solution with some stability property implies the existence of almost periodic solutions (cf. [4], [8], [10], [11], [12]).

Recently, Coppel [3], Yoshizawa [13, 14, 15] and Kato [7] have shown the existence of an almost periodic solution for ordinary differential equations and for functional differential equations without the assumption that the solution is unique. All of them required the existence of a bounded solution which is uniformly stable. Clearly, uniform stability implies the uniqueness of the bounded solution for initial value problem.

In this paper, more generally, we shall discuss functional differential equations with infinite retardation and the existence theorem for almost periodic solutions by assuming the existence of a bounded solution with some stability property which does not imply the uniqueness for initial value problem.

2. Hale's space and some lemmas. First, we shall give a class of Banach spaces considered by Hale [5]. Let $x = (x^1, x^2, \cdots, x^n)$ be any vector in R^n and let $||x||_{R^n} = \max_{1 \leqq i \leqq n} |x^i|$ be norm of x. Let $B = B((-\infty, 0], R^n)$ be a Banach space of functions mapping $(-\infty, 0]$ into R^n with norm $||\cdot||_B$. For any ϕ in B and any σ in $[0, \infty)$, let ϕ^σ be the restriction of ϕ to the interval $(-\infty, -\sigma]$. This is a function mapping $(-\infty, -\sigma]$ into R^n. We shall denote by B^σ the space of such functions ϕ^σ. For any $\eta \in B^\sigma$, we define the semi-norm $||\eta||_{B^\sigma}$ of η by

$$||\eta||_{B^\sigma} = \inf_{\phi} \{||\phi||_B : \phi^\sigma = \eta\} .$$

Then we can regard the space B^σ as a Banach space with norm $||\cdot||_{B^\sigma}$. If x is a function defined on $(-\infty, a)$, $a > 0$, then for each t in $[0, a)$ we define the function x_t by the relation $x_t(s) = x(t + s)$, $-\infty < s \leqq 0$. For numbers a and τ, $a > \tau$, we denote by A_τ^a the class of function x mapping $(-\infty, a)$ into R^n such that x is a continuous function on $[\tau, a)$ and $x_\tau \in B$. The space B is assumed to have the following properties: