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ON THE TANGENT SPHERE BUNDLE OF A RIEMANNIAN 2-MANIFOLD

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1. Introduction. Let V be an oriented Riemannian 2-manifold. The bundle $T_1(V)$ of the tangent unit vectors of V can be equipped with a family of natural Riemannian metrics given by the following line element:

$$d\sigma^2 = g_{ik}dx^i dx^k + \rho g_{ik}\delta_y{}^i \delta_y{}^k \ ,$$

where g_{ik} is the metric tensor of the basic manifold V, ρ is an arbitrary non-zero real constant and we have put

$$(1) \qquad g_{ik}y^i y^k = 1\ , \qquad \delta y^i = dy^i + \left\{ {i \atop j \quad k} \right\} y^j dx^k \ .$$

This metric in the case $\rho = 1$ was introduced and studied by S. SASAKI [2]. In a recent paper [1] W. KLINGEBERG and S. SASAKI investigated the tangent sphere bundle of a 2-sphere. The geometry of the tangent sphere bundle of a Euclidean 3-space was investigated by A. M. VASIL'EV in another approach [3].

In this paper we consider the tangent sphere bundle of an arbitrary Riemannian 2-manifold equipped with the generalized Sasaki-metric (1). We carry out our discussions using a special orthogonal frame: the first vector of the frame is the horizontal lift of the supporting element (i.e., of the regarded point of $T_1(V)$), the second and the third ones are the horizontal and vertical lifts of the normalized vector which is orthogonal to the supporting element.

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The author is indebted to A. Szücs (Budapest) for the verbal observation that if $(x(t), y(t))$ forms a geodesic in $T_1(M)$ then $y(t)$ moves on a simple helix relative to the parallel displacement necessarily (cf. Theorem 1).

3. The structure equations. The Riemannian connection of the