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THE GALOIS GROUP OF LOCAL FIELDS

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Introduction. Let Q_p be the p-adic number field, k a finite extension of Q_p, and G the Galois group of $\bar{k}/k$. In 1968, Jakovlev [1] described the structure of the Galois group G in the case of $p \neq 2$. In his paper, he determined the number of generators of G and gave a description of the relations among the generators. But, as he recognized himself there, those relations were considerably complicated and it is difficult to look through the structure of the Galois group only by those relations. In order to determine the structure of the Galois group up to isomorphism, as we shall show later, we need not the exact description of the relations, for we can characterize the relations among the generators by more general and loose conditions.

Let K be the maximal tamely ramified extension of Q_p and let

$$A = \mathrm{Gal.}(K/Q_p)\ , \quad B = \mathrm{Gal.}(\bar{Q}_p/K)\ .$$

Then the exact sequence

$$1 \longrightarrow B \longrightarrow G \longrightarrow A \longrightarrow 1$$

splits, and G is an holomorph extension of B by A. By fixing one splitting morphism $A \longrightarrow G$, we can consider A as a subgroup of G and B as a pro-p-group with the operator domain A. In § 6, we define "the projective envelope" P of the A-pro-p-group B, and consider the relation among the generators of B as an element of P. In § 7, we define the notion of ω-regularity for the elements of P, and prove that Ker.$(P \longrightarrow B)$ is generated by an ω-regular element of P. Furthermore, we prove that for any two ω-regular elements π, π' congruent to each other modulo certain normal subgroup of P, there is an automorphism σ of PA such that $\pi^\sigma = \pi'$, where PA is the holomorph extension of P by A.

By this fact, the problem of determination of the structure of the Galois group is entirely reduced to the problem of determination of certain non-degenerate skew symmetric quadratic form corresponding to the Galois group. Though we do not show here, we can easily classify such quadratic forms, and determine the one of them which corresponds to the Galois group.