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SOME THEOREMS ON (CA) ANALYTIC GROUPS II

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Abstract. An analytic group G is called (CA) if the group of inner automorphisms of G is closed in the Lie group of all (bicontinuous) automorphisms of G. It has been previously proved by this author that each non-(CA) analytic group G can be densely immersed in a (CA) analytic group H, such that the center of G is closed in H. We now show that there is no (CA) analytic group "smaller" than H into which G can be densely immersed, but H, however, is not the "smallest" such (CA) analytic group. Furthermore, we will isolate those properties of H which determine it uniquely up to dimension, diffeomorphism, diffeomorphism together with local isomorphism, and finally isomorphism.

1. Introduction. By an analytic group and an analytic subgroup of a Lie group, we mean a connected Lie group and a connected Lie subgroup, respectively. If G and H are Lie groups and φ is a one-to-one (continuous) homomorphism from G into H, φ will be called an immersion. φ will be called closed or dense, as $\varphi(G)$ is closed or dense in H. G_0 and $Z(G)$ will denote the identity component group and center of G, respectively.

If G is an analytic group, $A(G)$ will denote the Lie group of all (bicontinuous) automorphisms of G, topologized with the generalized compact-open topology. G will be called (CA) if $I(G)$, the Lie group of all inner automorphisms of G, is closed in $A(G)$. It is well known that G is (CA) if and only if its universal covering group is (CA).

If G is a normal analytic subgroup of an analytic group H, then each element h of H induces an automorphism of G, namely, $g \mapsto hgh^{-1}$. We will denote this homomorphism from H into $A(G)$ by ρ_{GH}. $I_H(h)$ will denote the inner automorphism of H determined by $h \in H$. More generally, if A is a subset of H, $I_H(A)$ will denote the set of all inner automorphisms of H determined by elements of A. $I_H(H)$ will be written as $I(H)$, and the mapping $h \mapsto I_H(h)$ of H onto $I(H)$ will be denoted by I_H.

If N is an analytic group and H is an analytic subgroup of $A(N)$, then $N \circledS H$ will denote the semidirect product of N and H. On the other hand, if G is an analytic group containing a closed normal analytic subgroup N and a closed analytic subgroup H, such that $G = NH$,