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THE REPRESENTATION THEOREM AND THE H^p SPACE THEORY ASSOCIATED WITH SEMIGROUPS ON LIE GROUPS

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Introduction. In [7] Stein has considered one-parameter semigroups of operators $\{T^t\}_{t\geqq 0}$ defined simultaneously on all spaces $L^p(G)$, $1 \leqq p \leqq \infty$, for a Lie group G, which satisfy the following properties:

(a) $\| T^t f\|_p \leqq \| f \|_p$.

(b) T^t is a self-adjoint operator on $L^2(G)$.

(c) $T^t f \geqq 0$ for $f \geqq 0$.

(d) $T^t 1 = 1$.

Leading examples of such a semigroup are the heat diffusion semigroup and the Poisson semigroup. Our purpose is to develop the analogue of results in classical harmonic analysis in the context of these semigroups.

In Section 2, we state the known facts about these semigroups, which are due to [2], [3], [5] and [7], and we prove the convergence theorems by using these facts.

In Section 3, we shall obtain results analogous to the classical properties of harmonic functions and subharmonic functions. The main result in this section is the representation theorem for harmonic functions. The basic tool which is used there is the maximum (minimum) principle for the heat equation and the Laplace equation on Lie groups.

In Section 4, we study the H^p space theory in a noncompact Lie group, analogous to classical one, which has been developed in Coifman and Weiss [1] for the case of a compact Lie group. In this section, to obtain an extension of the theorem of F. and M. Riesz, we apply the idea of a theorem concerning nontangential boundedness.

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1. Notations. In this section we fix notations which will be used in Sections 2-4. If S is a topological space, $C_b(S)$ denotes the set of all bounded continuous real valued functions on S. The set of all f in $C_b(S)$ which vanish at infinity is denoted by $C_0(S)$. The set of all f in