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NOTE ON GENERALIZED INFORMATION FUNCTION

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Abstract. Solution of a functional equation, connected with entropy, directed divergence, inaccuracy and their generalization of type β etc. is obtained without the assumption of any regularity condition on the functions involved.

1. Introduction. In [3], the following functional equation

$$f(x) + (1-x)^{\beta} g(y/(1-x)) = h(y) + (1-y)^{\beta} k(x/(1-y)) , \tag{1.1}$$

for $x, y \in [0, 1[$ with $x + y \in [0, 1]$, $\beta \neq 1$, was considered, giving the details of its association with entropy of type β [2], directed divergence of type β [9] and inaccuracy of type β [10]. For $\beta \to 1$, these measures reduce to Shannon's entropy [11], directed divergence [6] and inaccuracy [5].

In this paper, the functional equation (1.1) is solved by simple and direct method, without any further assumption on the functions, by reducing (1.1) to a similar form involving only one function,

$$u(x) + (1-x)^{\beta} u(y/(1-x)) = u(y) + (1-y)^{\beta} u(x/(1-y)) . \tag{1.2}$$

REMARK 1. If $u: [0, 1] \to R$ (reals) is a solution of (1.2), then $v(x) = u(x) - u(1)x^{\beta}$ is also a solution of (1.2) with $v(x) = v(1-x)$, that is, with v symmetric. Then the solution of (1.2) can be obtained from [1, 8], for $\beta \neq 1$. So, no generality is lost in considering the symmetric solution of (1.2). Thus the general solution of (1.2) for $\beta \neq 1$ is given by $u(x) = A[x^{\beta} + (1-x)^{\beta} - 1] + Bx^{\beta}$.

2. Solution of (1.1). Let $f, h: [0, 1[\to R$, $g, k: [0, 1] \to R$, satisfy the functional equation (1.1) for $x, y \in [0, 1[$ with $x + y \in [0, 1]$, where $\beta (\neq 2)$ is positive.

For $x = 0$, (1.1) gives

$$h(y) = g(y) + b_1(1-y)^{\beta} + b_2, \quad \text{for} \quad y \in [0, 1[, \tag{2.1}$$

where b_1, b_2 are constants.

With $y = 1 - x$ in (1.1), (1.1) becomes with the help of (2.1),

$$f(x) = g(1-x) + c_1 x^{\beta} + c_2(1-x)^{\beta} + b_2, \quad \text{for} \quad x \in]0, 1[, \tag{2.2}$$