Tôhoku Math. Journ.
30 (1978), 561-573.

INDEX OF SOME GAUSS-CRITICAL SUBMANIFOLDS

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(Received May 13, 1977, revised February 7, 1978)

Introduction. As is well-known, the Gauss map is an excellent device in classical differential geometry where curves and surfaces in a Euclidean three-space are studied. The same is true when the Gauss map is applied to an m-dimensional submanifold in a Euclidean n-space. In this case the image lies in the Grassmann manifold $G(m, n-m)$ which is not a sphere nor a projective space if m satisfies $1 < m < n-1$.

Let M be an m-dimensional compact orientable C^∞ submanifold in a Euclidean n-space E^n such that the Gauss map $\Gamma: M \to G(m, n-m)$ is regular. We consider only the case $1 < m < n-1$, for, if $m = n-1$, then Γ maps every closed hypersurface onto the $(n-1)$-sphere, whereas, if $m = 1$, a simpler method may be available. Nevertheless, one of the motives of the present study lies in the fact:

Let C be a closed curve with positive curvature in a Euclidean three-space. Then the Gauss image of C in the standard sphere has the least length when and only when C lies in a plane.

Assuming the standard Riemannian metric on $G(m, n-m)$, we get a volume form on $\Gamma(M)$. From the pull back of this volume form to M we get an integral $\mathrm{Vol}^*(\Gamma(M))$. As the Gauss image $\Gamma(M)$ is immersed in the Grassmann manifold, $\mathrm{Vol}^*(\Gamma(M))$ is not always the volume of $\Gamma(M)$. When M moves smoothly in E^n, $\mathrm{Vol}^*(\Gamma(M))$ moves in $\boldsymbol{R}$. Thus we can consider a submanifold M_0 which is a critical point of $\mathrm{Vol}^*(\Gamma(M))$. M_0 is called a Gauss-critical submanifold and is denoted by GCS. As it is always the case with critical points, there arises the problem of finding the index. The purpose of the present paper is to prove that a submanifold M which lies in a linear subspace E^{m+1} as a closed hypersurface with positive second fundamental form is a GCS whose index is zero.

A theorem related to this result has been obtained by Chern and Lashof [1], namely,

THEOREM OF CHERN AND LASHOF. *Let $i: M \to E^n$ be an immersion of an m-dimensional compact manifold M into a Euclidean n-space E^n. Then the total absolute curvature $\tau(M, i, E^n)$ is equal to 2 if and only*