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TRANSFORMATION OF CONDITIONAL WIENER INTEGRALS UNDER TRANSLATION AND THE CAMERON-MARTIN TRANSLATION THEOREM

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1. Introduction. Consider the Wiener measure space $(C[0, t], \mathfrak{W}^*, m_w)$ where $C[0, t]$ is the Wiener space consisting of all real valued continuous functions x on the interval $[0, t]$ in R^1 with $x(0) = 0$ for fixed $t \in (0, \infty)$; $\mathfrak{W}$ is the algebra of Borel cylinders in $C[0, t]$, i.e., the collection of all subsets W of $C[0, t]$ of the type

$$W = \{x \in C[0, t]; [x(s_1), \cdots, x(s_n)] \in B\} \tag{1.1}$$

where n is an arbitrary positive integer, $0 = s_0 < s_1 < \cdots < s_n \leqq t$, and B is an arbitrary member of the σ-algebra $\mathfrak{B}^n$ of the Borel sets in the n-dimensional Euclidean space R^n; m_w is a probability measure on the algebra $\mathfrak{W}$ defined for W as in (1.1) by

$$\begin{aligned} (1.2)\quad & m_w(W) \\ & = \left\{(2\pi)^n \prod_{j=1}^{n} (s_j - s_{j-1})\right\}^{-1/2} \int_B \exp\left\{-2^{-1} \sum_{j=1}^{n} (\xi_j - \xi_{j-1})^2 (s_j - s_{j-1})^{-1}\right\} m_L(d\xi) \end{aligned}$$

where $\xi = (\xi_1, \cdots, \xi_n) \in R^n$, $\xi_0 = 0$ and m_L is the Lebesgue measure on $(R^n, \mathfrak{B}^n)$; $\mathfrak{W}^*$ is the σ-algebra of Carathéodory measurable subsets of $C[0, t]$ with respect to the outer measure derived from the measure m_w on the algebra. Needless to say $\mathfrak{W}^*$ contains the σ-algebra $\sigma(\mathfrak{W})$ generated by $\mathfrak{W}$ and the Wiener measure space is a complete measure space. The $\mathfrak{W}^*$-measurability and m_w-integrability of a functional on $C[0, t]$ will be referred to as the Wiener measurability and Wiener integrability.

By a conditional Wiener integral we mean specifically the conditional expectation $E^w(Y|X)$ of a real or complex valued Wiener integrable functional Y conditioned by the functional X on the Wiener space defined by

$$X[x] = x(t) \qquad \text{for} \quad x \in C[0, t] \tag{1.3}$$

where the conditional expectation $E^w(Y|X)$ is not given as an equivalence class of random variables on the probability space $(C[0, t], \mathfrak{W}^*, m_w)$ but as an equivalence class of random variables on the probability space