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PROPERTY L AND W-* ALGEBRAS OF TYPE I[1]

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Abstract. Tpye I W-* algebras do not have property L.

Let $\mathscr{A}$ be a W-* algebra acting in separable Hilbert space h and let $\mathscr{U}(\mathscr{A})$ denote the unitary operators in $\mathscr{A}$. Corollary I.5.10 of [3] states that $\mathscr{A}$ has direct integral decomposition into factors given by $\mathscr{A} = \int_\Lambda \oplus \mathscr{A}(\lambda)\mu(d\lambda)$. This paper assumes the reader is familiar with [4] and Chapter I of [3].

DEFINITION. $\mathscr{A}$ has *property* L if there is a sequence $\{U_n\}$ contained in $\mathscr{U}(\mathscr{A})$ such that $\{U_n\} \to 0$ weakly and such that $\{U_n A U_n^*\} \to A$ strongly for each $A \in \mathscr{A}$.

Property L is a partial form of commutivity that was introduced by Pukánszky in [2]. We shall use direct integral theory to show that no type I W-* algebra has property L.

We establish some notation before proving two essential lemmas. $\mathscr{A}'$ denotes the commutant of $\mathscr{A}$ and is also a W-* algebra. By the center of $\mathscr{A}$, we mean the abelian W-* algebra $\mathscr{Z}(\mathscr{A}) = \mathscr{A} \cap \mathscr{A}'$. $\mathscr{A}_1$ represents the unit ball of $\mathscr{A}$ and h_∞ denotes the underlying Hilbert space of h, i.e., $h = \int_\Lambda \oplus h_\infty \mu(d\lambda)$ (cf. [3] Definition I.2.4).

LEMMA 1. *Let* $\mathscr{A} = \int_\Lambda \oplus \mathscr{A}(\lambda)\mu(d\lambda)$ *be a* W-* *algebra acting in* h *and let* S *denote* $B(h_\infty)_1$ *taken with the strong-* operator topology. Then if* N *is a Borel subset of* Λ, *the set* $F = \{(\lambda, T) \mid \lambda \in N, T \in \mathscr{A}(\lambda) \cap S\}$ *is a Borel subset of* $\Lambda \times S$.

PROOF. By [3] Lemma I.4.11, S is a complete separable metric space. Let d denote the metric which defines the topology on S. By [4] Lemma 1.5(a, c), there is a countable sequence of disjoint closed subsets e_i of Λ such that if $e = \Lambda - \bigcup_{i=1}^\infty e_i$, then $\mu(e) = 0$ and there is

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