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# LEAVES WITH NON-EXACT POLYNOMIAL GROWTH

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**1. Introduction.** Let $\mathscr{F}$ be a codimension one foliation of class $C^r$ $(r \geqq 0)$ of a compact manifold $M$ which is tangent to the boundary. Let $M = \bigcup_{i=0}^{m} U_i$ be a finite cover of $M$ by regular distinguished charts. Let $F$ be a leaf and $P_0 \subset F \cap U_0$ a plaque contained in $F$. The *growth function* of $F$ at $P_0$, $f: \boldsymbol{Z}^+ \to \boldsymbol{Z}^+$ is defined by

$f(n) =$ the number of distinct plaques which can be reached from the initial plaque $P_0$ by a plaque chain of length at most $n$ (see [5], [6]).

DEFINITION. $F$ has *polynomial growth* if the growth function $f$ of $F$ is dominated by a polynomial. In this case we define the *upper growth* and *lower growth* of $F$, denoted $u.gr(F)$ and $l.gr(F)$ respectively, as follows:

$$u.gr(F) = \inf\{d \in \boldsymbol{R}^+ \mid f \text{ is dominated by the polynomial } g(n) = n^d\}$$
$$l.gr(F) = \sup\{d \in \boldsymbol{R}^+ \mid f \text{ dominates the polynomial } g(n) = n^d\}\ .$$

Finally, we say $F$ has *exact polynomial growth* of degree $d$ if the growth function of $F$ dominates a polynomial of degree $d$ and is dominated by a polynomial of degree $d$.

It is easy to see that the upper and lower growth of a leaf depend neither on the choice of regular distinguished charts nor on the choice of the initial plaque ([5]).

In [4] Hector posed the following problem: if $F$ has polynomial growth, then does $F$ have exact polynomial growth of an integer degree? In this paper we give two partial answers to this problem. We remark that the answer is affirmative if the foliation is transversely analytic or it is almost without holonomy ([7]).

THEOREM A. *Let $\mathscr{F}$ be a codimension one foliation of class $C^r$ $(r \geqq 2)$ of a compact manifold $M$ which is tangent to the boundary. Let $F$ be a leaf of $\mathscr{F}$. Assume that the growth function of $F$ is dominated by a polynomial of degree* 2. *Then $F$ has exact polynomial growth of degree* 0, 1 *or* 2.