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ALMOST PERIODIC GROSS-SUBSTITUTE DYNAMICAL SYSTEMS

Dedicated to Professor Taro Yoshizawa on his sixtieth birthday

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1. Introduction. In this note we shall study tâtonnement processes with time-dependent almost periodic coefficients. The model process is given by a system of ordinary differential equations

$$\frac{dp_i}{dt} = \lambda_i E_i(p, t), \quad i = 1, 2, \cdots, n \tag{0}$$

where $p = (p_i)$ is a price-vector, $E_i(p, t)$ is the excess demand function for the ith good and λ_i is a positive constant. These equations form a mathematical model for the classical law of supply and demand. We shall assume below that the system (0) is a gross-substitute system that satisfies Walras' law and that $E(p, t)$ is almost periodic in t. An example of the system we consider is given by $\lambda_i = 1$ for all i and

$$E_i(p, t) = \left(\sum_{\alpha=1}^{M} \sum_{j=1}^{n} a_{ij}^{\alpha}(t) p_j^{\alpha}\right) \Big/ p_i ,$$

where a_{ij}^{α} is almost periodic in t, $a_{ij}^{\alpha} \geqq 0$ when $i \neq j$ and $\sum_{i=1}^{n} a_{ij}^{\alpha}(t) \equiv 0$ for all j and α.

Autonomous tâtonnement processes have been studied extensively in the econometrica literature, cf. [8, 10] for example. The stability and limiting behavior of these systems is well understood, cf. [1-3, 6, 8, 10, 12]. However if one wishes to build a theory of such economic models which reflects changes due to seasonal adjustments, then it is important to study time-dependent or nonautonomous systems. The theory we describe here is adequate to describe the limiting behavior of systems with almost periodic seasonal adjustments. In the example above such systems would occur if the coefficients $a_{ij}^{\alpha}(t)$ are periodic with incommensurable periods, cf. [5, 13].

This paper is a generalization of the periodic theory presented in Nakajima [7]. In particular we will show that any "positively compact" solution of (0) is asymptotically almost periodic, cf. [13]. As we shall

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