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REMARKS ON THE LIMIT SETS OF KLEINIAN GROUPS

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1. The so-called combination theorems of Maskit play an important role in the theory of Kleinian groups. In [6], Maskit proved that every function group can be constructed from elementary groups, quasi-Fuchsian groups and degenerate groups by using his combination theorems. Moreover, in [1] Abikoff and Maskit proved that every finitely generated Kleinian group can be constructed from elementary groups, degenerate groups and web groups in a similar manner. In this note we investigate the limit sets of Kleinian groups which are constructed by using the combination theorems.

2. Let G be a Kleinian group and denote by $\Omega(G)$ and $\Lambda(G)$ the region of discontinuity and the limit set of G, respectively. We denote by SL$'$ the group of all the Möbius transformations. Consider a sequence $\{C_n\}$ of Jordan curves on $\hat{C}$ and a point $z \in \hat{C}$. We say that $\{C_n\}$ nests about z, if C_{n+1} separates z from C_n for every natural number n and if the sequence of spherical diameters of $\{C_n\}$ forms a null sequence.

Let C be a Jordan curve on $\hat{C}$ and $\{g_n\}$ be a sequence of elements of SL$'$. We say that the sequence $\{g_n(C)\}$ converges to a point $z \in \hat{C}$, if there exists a point $x \in C$ so that $\{g_n(x)\}$ converges to z and the sequence of spherical diameters of $\{g_n(C)\}$ forms a null sequence.

3. Let G be a Kleinian group and let H be a subgroup of G. A subset S on $\hat{C}$ is called precisely invariant under H in G, if $h(S) = S$ for every $h \in H$ and $g(S) \cap S = \varnothing$ for every $g \in G - H$. For a cyclic subgroup H of G, a precisely invariant disc B for H is the interior of a closed topological disc $\bar{B}$ on $\hat{C}$, where $\bar{B} - \Lambda(H)$ is precisely invariant under H in G and $\bar{B} - \Lambda(H) \subset \Omega(G)$.

We use the combination theorems in the following forms.

COMBINATION THEOREM I. *Let G_1 and G_2 be two Kleinian groups and let B_i $(i = 1, 2)$ be a precisely invariant disc under H, a finite or a parabolic cyclic subgroup of both G_1 and G_2. Assume that B_1 and B_2 have the common boundary γ and $B_1 \cap B_2 = \varnothing$. Let G be the group generated by G_1 and G_2. Then we have the following:*