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# LIAPUNOV'S SECOND METHOD IN FUNCTIONAL DIFFERENTIAL EQUATIONS

Dedicated to Professor Taro Yoshizawa on his sixtieth birthday

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**1. Introduction.** For the theory of stability in differential equations, Liapunov's second method may be the most important. In the case of functional differential equations, there were also many attempts to establish various kinds of Liapunov type theorems, see e.g. [1]~[6], [8], [10]~[18]. Among them, there are three main ideas, one based on Liapunov functionals (cf. [13]), one by Razumikhin [15] and one by Barnea [1]. The idea based on Liapunov functionals is the most perfect in the sense that we can develop it in the way parallel to that for ordinary differential equations including the establishment of necessary and sufficient conditions of stability. However, the construction of such a Liapunov functional is very hard for concrete problems. This difficulty stimulates the development of the ideas by Razumikhin and by Barnea (cf. [2], [5], [6], [14], [17], [18]). The author also gave some extensions of the ideas in a unified way combining both, see [10], [11]. In this paper we give several results in the same direction. For the examples, refer to [12].

Recently, Burton [3] presented a kind of stability theorems. Noting that actually his result is deeply related with the choice of the phase space and that in a concrete problem there are several possibilities for the choice of the phase space, we state our results for functional differential equations on an abstract phase space discussed in [9].

**2. Admissible phase space.** Let $(X, \|\cdot\|_X)$, or simply $X$, be a linear space of $R^n$-valued functions on $(-\infty, 0]$ with a semi-norm $\|\cdot\|_X$, and denote by $X_\tau$ the space of functions $\phi(s)$ on $(-\infty, 0]$ which are continuous on $[-\tau, 0]$ and satisfying $\phi_{-\tau} \in X$ for given $X$ and $\tau \geqq 0$, where and henceforth $\phi_t$ denotes the function on $(-\infty, 0]$ defined by $\phi_t(s) = \phi(t+s)$.

The space $(X, \|\cdot\|_X)$ is said to be *admissible*, if the following are satisfied: For any $\tau \geqq 0$ and any $\phi \in X_\tau$

($a_1$) $\phi_t \in X$ for all $t \in [-\tau, 0]$, especially, $\phi_0 = \phi \in X$,

($a_2$) $\phi_t$ is continuous in $t \in [-\tau, 0]$,