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ON HELICES AND MULTIPLE WIENER INTEGRALS OF A GAUSSIAN AUTOMORPHISM

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0. The purpose of this note is to represent helices of a Gaussian automorphism by the multiple Wiener integrals and to calculate the multiplicity of helices.

1. Let $(\Omega, \mathscr{F}, P)$ be a complete separable probability space and $(T, \mathscr{F}_0)$ a system on Ω, that is, a pair of an automorphism of Ω and a complete sub-σ-field of $\mathscr{F}$ such that

(a) $\bigvee_{n=-\infty}^{\infty} T^n\mathscr{F}_0 = \mathscr{F}$,

(b) $T\mathscr{F}_0 \supset \mathscr{F}_0$.

Let $H = L_0^2(\Omega)$ denote the Hilbert space of all squarely integrable real random variables with zero-expectations and H_n the subspace of H consisting of all elements measurable with respect to $T^n\mathscr{F}_0$ for each n.

DEFINITION 1. A process $X = (x_n)$ is called a helix for a system $(T, \mathscr{F}_0)$ if the following conditions are satisfied:

(a) $x_0 = 0$,

(b) $x_n - x_m \in H_n \ominus H_m$ for all m and n with $m < n$,

(c) $(x_n - x_m) \circ T^{-1} = x_{n+1} - x_{m+1}$ for all m and n.

By the condition (b), $(x_n, T^n\mathscr{F}_0)_{n \geqq 0}$ can be regarded as a square-integrable martingale and further by the condition (c), all x_n can be written as

$$x_n = \sum_{k=1}^{n} x \circ T^{-(k-1)}$$

for some $x \in H_1 \ominus H_0$.

DEFINITION 2. For helices $X = (x_n)$ and $X' = (x'_n)$, $\mu_{\langle X, X' \rangle}$ denotes the signed measure on $(\Omega, \mathscr{F}_0)$ such that

$$d\mu_{\langle X, X' \rangle} = E[x_1 x'_1 \mid \mathscr{F}_0] dP .$$

If $\mu_{\langle X, X' \rangle}$ is a null measure, we say that X and X' are strictly orthogonal. If $X = X'$, then $\mu_{\langle X, X \rangle}$ is denoted simply by $\mu_{\langle X \rangle}$.

By the martingale property of helices, we can define the following which is similar to the martingale-transform: