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A MULTILINEARIZATION OF LITTLEWOOD-PALEY'S g-FUNCTION AND CARLESON MEASURES

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Introduction. Recently Coifman and Meyer [4] introduced a class of multilinear operators as a multilinearization of Littlewood-Paley's g-function. They studied L^2 estimates of such operators, using the notion of Carleson measures. In this note we shall develop their study further, by weakening their assumptions and obtain H^1, BMO and L^p estimates. Our techniques are essentially modifications of theirs, but we need many devices to make their ideas deeper at many points. Our main results are Theorems 1 and 2, and stated in Section 2. Notations and definitions are given in Section 1. There we introduce some classes of weight functions to state our theorems. In Section 3 we shall give preliminary lemmas and prove the main theorems in Section 4. In these sections Carleson measures play very important roles, but there we only quote lemmas giving relations between BMO and Carleson measures. We shall treat them systematically in Section 6, because we wish to treat many things related to BMO and Carleson measures. There, for example, we shall improve some recent results of Strichartz [11]. Some applications and examples of the main theorems are given in Section 5.

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1. Notations and Definitions. $\mathscr{D} = \mathscr{D}(\boldsymbol{R}^n) = C_0^\infty(\boldsymbol{R}^n)$ denotes the set of all infinitely differentiable functions with compact support on $\boldsymbol{R}^n$: the n-dimensional Euclidean space. $\mathscr{S} = \mathscr{S}(\boldsymbol{R}^n)$ is the set of all infinitely differentiable functions whose derivatives decrease rapidly. Recall that a locally integrable function f is said to be of bounded mean oscillation on $\boldsymbol{R}^n$ if the mean oscillation of f on any cube Q with sides parallel to the axes

$$\mathrm{MO}\,(f, Q) = \frac{1}{|Q|}\int_Q |f(x) - f_Q|dx$$

is uniformly bounded, where f_Q denotes the mean of f on Q

$$f_Q = \frac{1}{|Q|}\int_Q f(x)dx$$