

## THE NISHIMORI DECOMPOSITIONS OF CODIMENSION-ONE FOLIATIONS AND THE GODBILLON-VEY CLASSES

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(Received June 23, 1981)

**Introduction.** It seems to be an interesting problem to study the relations between the geometric behaviour of leaves in codimension-one foliations and their characteristic classes. The first important result in this direction was that of Herman [Her]. He proved that the Godbillon-Vey invariants of foliations by planes vanish. This result was generalized by Morita and Tsuboi to the case of foliations without holonomy ([Mo-T], see also [Mi-T]).

Recently, two remarkable results in this line were obtained. First, Nishimori looked at a certain class of codimension-one foliations (of finite depth and with abelian holonomy in his terminology [Ni 2, 3]), and saw that such foliations admit nice decompositions—so-called SRH-decompositions. By using the decomposition, he proved that the Godbillon-Vey numbers of such foliations are zero if the dimension of the foliated manifold is three [Ni 4].

Secondly, there has been another class of foliations whose qualitative natures were fairly well-known. Recall that a codimension-one foliation is called almost without holonomy if the holonomy group of each non-compact leaf is trivial. For such foliations there is a structure theorem due mainly to Hector [Hec 1] and Imanishi [Im 2]. Let  $U$  be a connected component of the complement of the union of compact leaves in a foliation almost without holonomy. There is a homomorphism, called the Novikov transformation, from the fundamental group of  $U$  to the group of diffeomorphisms of the real line which describes the qualitative behaviour of each leaf in  $U$  ([No], [Im 1, 2, 3], [T 2]). Mizutani, Morita and Tsuboi defined the notion of foliated  $J$ -bundles in order to relate this homomorphism to the holonomy groups of compact leaves in the boundary of  $U$  and thus to treat functorially foliations almost without holonomy. They proved that the Godbillon-Vey classes of such foliations are all trivial [M-M-T].

Our goal in this paper is to enlarge the above list of foliations with trivial characteristic classes. We consider the class of codimension-one foliations which satisfy the following conditions (P) and (A):