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DEFINING IDEALS OF THE CLOSURES OF THE CONJUGACY CLASSES AND REPRESENTATIONS OF THE WEYL GROUPS

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1. **Introduction.** Let G be a connected reductive algebraic group over the complex number field $\boldsymbol{C}$ and T be its maximal torus. We denote the Lie algebras of G and T by $\mathfrak{g}$ and $\mathfrak{t}$, respectively. Let O_x be the G-orbit containing $x \in \mathfrak{g}$ under the adjoint action of G on $\mathfrak{g}$. Then the Weyl group W of (G, T) naturally acts on the coordinate ring $\boldsymbol{C}[\mathfrak{t} \cap \bar{O}_x]$ of the scheme-theoretic intersection of $\mathfrak{t}$ and the Zariski closure $\bar{O}_x$ of O_x. We consider the following problem due to Kostant, Kraft, DeConcini and Procesi. (See [1] and [5].)

PROBLEM. *Describe* $\boldsymbol{C}[\mathfrak{t} \cap \bar{O}_x]$ *as a W-module for each nilpotent orbit* O_x *in* $\mathfrak{g}$.

When x is regular nilpotent, $\bar{O}_x$ is just the variety N consisting of all the nilpotent elements in $\mathfrak{g}$, and $\boldsymbol{C}[\mathfrak{t} \cap N]$ is isomorphic to the regular representation of W (Cf. Kostant [4].).

DeConcini and Procesi [1] have shown that for $G = GL(n, \boldsymbol{C})$, $\boldsymbol{C}[\mathfrak{t} \cap \bar{O}_x]$ is isomorphic to the representation induced from the trivial representation of a certain subgroup of parabolic type. They also naturally identified $\boldsymbol{C}[\mathfrak{t} \cap \bar{O}_x]$ with a certain representation of W constructed by Springer [11], [12] (Cf. §2 and §3 below for precise statements.). In [1] they conjectured that certain explicitly constructed polynomials form a generator system of the defining ideal of the variety $\bar{O}_x$ and proved the above results using these polynomials.

In this note we first give another candidate for a generator system of the defining ideal of $\bar{O}_x$ and show that the proof of the results in [1] can be a little simplified by replacing their polynomials by ours (§2, §3). Though some of the statements and the arguments in §2 and §3 are similar to those in [1], we include them for convenience of the readers.

For a general reductive group G the structure of $\boldsymbol{C}[\mathfrak{t} \cap \bar{O}_x]$ is not yet clear. We secondly show that for a nilpotent orbit of a certain type

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