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ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF ORDINARY DIFFERENTIAL EQUATIONS

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1. Introduction. The main purpose of this paper is to investigate the asymptotic behavior of solutions of the nonlinear differential equation on $[0, \infty) \times Q$,

$$\dot{x} = f(t, x) \ , \tag{1}$$

where Q is an open subset of R^n and $f(t, x)$ is continuous on $[0, \infty) \times Q$. Consider the following assumptions with respect to the equation (1): There exist nonnegative continuous functions $V(t, x)$ and $W(x)$ such that $V(t, x)$ is locally Lipschitzian with respect to x and $\dot{V}_{(1)}(t, x) \leqq -W(x)$ for $t \geqq 0$, $x \in Q$. Then it is well known that each bounded solution of (1) approaches the set $E = \{x \in Q: W(x) = 0\}$ as $t \to \infty$ under the assumption that $f(t, x)$ is bounded when x is bounded [7], [8], [9], [11]. Recently, LaSalle [4] obtained the same result under the weaker assumption that $f(t, x)$ satisfies Condition (B) (see Remark 1 below). In this paper, we analyze the problem posed above under a further relaxed assumption, Condition (C) below.

As an application, we shall investigate the asymptotic behavior of solutions of the second order scalar nonlinear differential equation

$$\ddot{x} + h(t, x, \dot{x})|\dot{x}|^{\alpha}\dot{x} + f(x) + g(t, x, \dot{x}) + p(t, x, \dot{x}) = 0 \ ,$$

where $\alpha \geqq 0$. In the case $\alpha = 0$, Onuchic [7], [8], [9] obtained sufficient conditions under which every solution, together with its derivative, tends to zero as $t \to \infty$. Since he applied the invariance principle, one of his most essential assumptions is the following: $h(t, x, y)$ is bounded when $x^2 + y^2$ is bounded. Many authors discussed the problem of relaxing the boundedness condition on h. One of these conditions is the growth condition on h. Thurston and Wong [10], Artstein and Infante [1] and others discussed this problem under the growth condition.

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2. Notation, definition and preparatory lemmas. We denote by