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APPROXIMATION OF OPERATOR SEMIGROUPS OF OHARU'S CLASS $(C_{(k)})$

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In [13], Takahashi and Oharu have proved a convergence theorem for sequences of semigroups of Oharu's classes $(C_{(k)})$ which extends the "continuous" Trotter theorem [15]. We establish a discrete version of this result (Theorem 1), i.e., with the approximating sequence of semigroups replaced by a sequence of powers of bounded linear operators. This is applied to two particular sets of $(C_{(1)})$-semigroups, one originating in the theory of approximation with exponential orders [7], the other coming from a discretization of a certain Cauchy problem [12], [13; Ex. 5.4]. Instead of a semi-discrete version of the latter problem we now consider its full discretization and show, e.g., that the weakened stability condition of Theorem 1 applies in this case. Moreover it will be shown that the two examples are not of class (A), in general, and a summary of further approximation properties will be given.

1. A discrete convergence theorem for semigroups of class $(C_{(k)})$. The classes $(C_{(k)})$ of semigroups have been introduced by Oharu [11; p. 250] in connection with abstract Cauchy problems. They have been investigated futher in [13]. To define them we need the following notations.

Let X be a Banach space, $[X]$ the space of bounded linear operators from X into itself, and $\{T(t),\ t > 0\}$ a semigroup of operators in $[X]$. Let $\omega_0 = \lim_{t\to\infty} t^{-1} \log \| T(t) \|$ be the type of the semigroup, and $\Sigma = \{f \in X;\ \lim_{t\to 0+} \| T(t)f - f \| = 0\}$ its continuity set. Supposing $\{T(t),\ t > 0\}$ to be strongly continuous on $(0, \infty)$, we denote by $R_0(\lambda)$ the Laplace transform

$$R_0(\lambda)f = \int_0^\infty e^{-\lambda t} T(t) f dt \quad (\lambda \in \boldsymbol{C}) \tag{1.1}$$

whenever the right hand side exists as a Bochner integral. Let A_0 be the infinitesimal operator of the semigroup and set $X_0 = \bigcup_{t>0} T(t)(X)$.

Following Oharu [11] one says that the semigroup belongs to class $(C_{(k)})$ if it satisfies the following conditions:

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