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INFINITESIMAL AUTOMORPHISMS AND SECOND VARIATION OF THE ENERGY FOR HARMONIC FOLLATIONS*

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Introduction. In [KT2] [KT3] we considered an energy functional for foliations $\mathscr{F}$ on a smooth compact oriented manifold, defined with respect to a Riemannian metric g_M on M. Harmonic Riemannian foliations were then characterized as critical foliations for this functional under an appropriate class of so called special variations (the relevant concepts are repeated in Section 2 of the present paper). The obvious analogy with the harmonic map theory of Eells and Sampson [ES] was the guiding principle.

In this paper we discuss the effect of curvature properties in the normal bundle of a Riemannian foliation $\mathscr{F}$ on:

(A) the existence of deformations of $\mathscr{F}$ through harmonic foliations;

(B) the existence of infinitesimal metric automorphisms of $\mathscr{F}$.

The curvature properties in the normal bundle of a Riemannian foliation (R-foliation) are described as follows. There is a unique metric and torsionfree connection ∇ in the normal bundle. Thus its curvature operator R_∇ is canonically attached to $\mathscr{F}$. The natural vanishing properties of R_∇ allow to view it as a skew-symmetric operator $R_\nabla(\mu, \nu)$ on sections $\mu, \nu \in \Gamma Q$ with values in the bundle $\mathrm{End}(Q)$. If $\mathscr{F}$ is viewed as modelled on a Riemannian manifold N by a Haefliger cocycle with isometric transition functions, then R_∇ is the curvature of the canonical connection on N pulled back by the local submersions with target N defining $\mathscr{F}$. The Ricci operator $\rho_\nabla: Q \to Q$ of $\mathscr{F}$ is then defined in terms of R_∇ by the usual formula (see (1.6) or (1.7) below).

We prove the following result.

THEOREM A. *Let $\mathscr{F}$ be a Riemannian foliation on a compact and oriented manifold M. Let g_M be a bundle-like metric on M in the sense of Reinhart* [R], *and assume $\mathscr{F}$ to be harmonic with respect to g_M. Assume the Ricci operator ρ_∇ of $\mathscr{F}$ to be $\leqq 0$ everywhere, and < 0 for at least one point $x \in M$. Then*:

(i) *there is no special variation of $\mathscr{F}$ through harmonic foliations*;

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