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BILINEAR FOURIER MULTIPLIERS

Dedicated to Professor Tamotsu Tsuchikura on his sixtieth birthday

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Introduction. An m-linear operator $T_\sigma(f_1, f_2, \cdots, f_m)$ is said to be an m-linear Fourier multiplier with symbol $\sigma(\xi_1, \xi_2, \cdots, \xi_m)$, if it has the following form

$$T_\sigma(f_1, f_2, \cdots, f_m) = (2\pi)^{-nm} \int_{R^{nm}} e^{ix\cdot(\xi_1+\cdots+\xi_m)}\sigma(\xi_1, \cdots, \xi_m)\hat{f}_1(\xi_1) \cdots \hat{f}_m(\xi_m)d\xi_1 \cdots d\xi_m ,$$

where $x, \xi_1, \cdots, \xi_m \in \boldsymbol{R}^n$, $x \cdot y = x_1y_1 + \cdots + x_ny_n$ and $\hat{f}$ is the Fourier transform of f, i.e. $\hat{f}(\zeta) = \int_{R^n} e^{-ix\cdot\zeta}f(x)dx$, which is denoted also by $\mathscr{F}f$. In [6] we showed the following: If $\sigma \in C^{2nm+1}(\boldsymbol{R}^{nm}\backslash\{0\})$ and $|\partial_\xi^\alpha\sigma(\xi)| \leqq C_\alpha|\xi|^{-|\alpha|}$, $\xi \neq 0$, $|\alpha| \leqq 2nm + 1$, then for $p_j \in [1, \infty]$ $(1 \leqq j \leqq m)$ and $0 \leqq 1/p = 1/p_1 + \cdots + 1/p_m \leqq 1$ it holds that

$$\| T_\sigma(f_1, \cdots, f_m)\|_p \leqq C(p_1, \cdots, p_m)\|f_1\|_{p_1} \cdots \|f_m\|_{p_m} , \tag{0.1}$$

$(f_j \in \mathscr{S}, j = 1, \cdots, m)$, where $\|f_j\|_{p_j} = \|f_j\|_{H^1}$ and $f_j \in H_{00}^1$ if $p_j = 1$, and in case $p = \infty$ the norm on the left-hand side is the BMO norm, which is denoted by $\|f\|_*$. Here $\mathscr{S}$ is the Schwartz class of smooth and rapidly decreasing functions, and H_{00}^1 is the space of all $f \in \mathscr{S}$ such that $\hat{f}$ has compact support bounded away from the origin. $\|f\|_p$ is the usual $L^p(\boldsymbol{R}^n)$ norm $(1 \leqq p \leqq \infty)$. See [7], [5] or [6] for the definitions of H^p spaces and BMO.

We say that T_σ has *Property* (C) if the above inequalities (0.1) hold for all $1 \leqq p_j \leqq \infty$ and $0 \leqq 1/p = 1/p_1 + \cdots + 1/p_m \leqq 1$. In this note we try to relax the assumption for σ to obtain Property (C) in the case $n = 1$ and $m = 2$. That is, we deal with bilinear Fourier multipliers with non-smooth symbols. Our symbols have singularities on finite pieces of rays issuing from the origin. Our prototype is Calderón's commutator K_1 in Section 2. It has singularities on the rays $\{\theta = \pm\pi/2\}$ and $\{\theta = (1\pm2)\pi/4\}$. In Section 1 we deal with non-homogeneous symbols.

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