Tôhoku Math. Journ.
36 (1984), 401-406.

STABILITY OF A MECHANICAL SYSTEM WITH UNBOUNDED DISSIPATIVE FORCES

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(Received September 3, 1983)

In this article we shall be concerned with a mechanical system described by the Lagrangian equation

$$\text{(1)} \qquad \frac{d}{dt}\frac{\partial T}{\partial \dot{q}} - \frac{\partial T}{\partial q} = -\frac{\partial \Pi}{\partial q} - B(t, q)\dot{q} + G(t, q)\dot{q} \ ,$$

with generalized coordinates $q \in R^n$ and generalized velocities $\dot{q} \in R^n$. Salvadori [5] gave sufficient conditions under which the equilibrium of (1) is asymptotically stable in the case where B and G are time-independent. Recently, Hatvani [2] gave the conditions of the (partial) asymptotic stability and instability for more general systems. To obtain a result of the asymptotic stability, he considered some familiar conditions and furthermore, the following:

(*) For any compact subset L of R^n ,

$$\gamma_L(t) := \sup\{\|G(t, q) - B(t, q)\| : q \in L\} \in F \ ,$$

where F is the set of all measurable functions $\xi(t) = \xi_1(t) + \xi_2(t)$, ξ_1, ξ_2: $[0, \infty) \to [0, \infty)$, such that ξ_1 is bounded on $[0, \infty)$ and $\int_0^\infty \xi_2(t)dt < \infty$. If $B(t, q) \equiv tE$ (E is the unit matrix in $R^{n\times n}$) and $G(t, q) \equiv 0$, however, the condition (*) does not hold. In this article, by employing the manner developed in [3] we shall overcome this difficulty for the dissipation B which is unbounded. That is, we shall show that the equilibrium $q = \dot{q} = 0$ of (1) is weakly uniformly asymptotically stable under some familiar conditions and the following; for any bounded continuous function $\psi(s)$ on $[0, \infty)$ there exist a sequence of positive numbers $\{s_n\}$ and a positive constant d, $s_{n+1} \geqq s_n + d$, such that $\operatorname{tr} B(s, \psi(s)) \not\equiv 0$ on $[s_n, s_n + d]$ for all n and that

$$\sum_{n=1}^{\infty}\left[\int_{s_n}^{s_n+d} \operatorname{tr} B(s, \psi(s))ds\right]^{-1} = \infty \ ,$$

where $\operatorname{tr} B(s, \psi(s))$ denotes the trace of $B(s, \psi(s))$. Thus, our result is applicable to a mechanical system with unbounded B satisfying $0 < \operatorname{tr} B \leqq Mt \cdot \log(1 + t) + N$, $t \geqq 0$, for some positive constants M and N. In