Tôhoku Math. Journ.
37 (1985), 79-93.

ON EXTREMAL QUASICONFORMAL MAPPINGS COMPATIBLE WITH A FUCHSIAN GROUP WITH A DILATATION BOUND

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(Received December 23, 1983, revised June 1, 1984)

1. **Introduction.** Let U be the upper half-plane and let Γ be a Fuchsian group. That is, Γ is a discrete subgroup of the real Möbius group $PSL(2, \boldsymbol{R})$, possibly consisting of only the identity transformation of $PSL(2, \boldsymbol{R})$. Let $L_\infty(\Gamma)$ be the closed linear subspace of $L_\infty(U)$ consisting of those $\nu \in L_\infty(U)$ which satisfy

$$(1.1) \qquad \nu(\gamma(w))\overline{\gamma}'(w)/\gamma'(w) = \nu(w) \quad \text{for every} \quad \gamma \in \Gamma\ .$$

We denote by $M(\Gamma)$ the open unit ball of $L_\infty(\Gamma)$. For ν in $M(\Gamma)$, we denote by $z = F_\nu(w)$ the uniquely determined automorphism of U which is a generalized solution in U of the Beltrami equation $F_{\bar{w}} = \nu F_w$ and which leaves 0, 1, ∞ fixed. The mapping F_ν is called the normalized quasiconformal automorphism of U with complex dilatation $\nu = \nu(w)$ (see Lehto and Virtanen [12, p. 185 and p. 194]). As is known, F_ν is extensible to a homeomorphism of the closure $\bar{U} = U \cup \hat{R}$ of U in the extended complex plane $\hat{C}$, which we denote by the same letter F_ν.

Let σ be a Γ-invariant closed subset of the extended real line $\hat{R}$, which contains 0, 1 and ∞. Let E be a Γ-invariant measurable, possibly empty, subset of U such that the closure of E/Γ in $\{U \cup (\hat{R} \setminus \sigma)\}/\Gamma$ is a compact proper subset of $\{U \cup (\hat{R} \setminus \sigma)\}/\Gamma$. Let $b(w)$ be a non-negative bounded measurable function on E, being automorphic for Γ and satisfying

$$(1.2) \qquad 0 \leqq c_1 = \operatorname*{ess\,sup}_{w \in E} b(w) < 1\ .$$

Let $D = U \setminus E$. By the above property of E, we easily see that the set D/Γ has a positive measure. For ν in $M(\Gamma)$, we put

$$(1.3) \qquad K(\nu|_D) = (1 + \|\nu|_D\|_\infty)/(1 - \|\nu|_D\|_\infty)\ ,$$

where $\|\nu|_D\|_\infty$ means the L_∞ norm of the restriction $\nu|_D$ of ν to D. In the case E is empty, we use the notation $K(\nu)$ instead of $K(\nu|_U)$.

Suppose that μ is a prescribed element in $M(\Gamma)$ satisfying $|\mu(w)| \leqq b(w)$ a.e. in E. We consider the class $M_\mu \equiv M_\mu(\Gamma, \sigma, E, b)$ consisting of those $\nu \in M(\Gamma)$ which satisfy the conditions $F_\nu|_\sigma = F_\mu|_\sigma$ and $|\nu(w)| \leqq b(w)$ a.e. in E. We put