Tôhoku Math. J.
40 (1988), 165-187.

FOLIATIONS AND SUBSHIFTS

Dedicated to Ky Fan on his retirement

JOHN CANTWELL[1] AND LAWRENCE CONLON[2]

(Received November 4, 1986)

Introduction. Let $(M, \mathscr{F})$ be a closed, transversely orientable, C^2-foliated manifold of codimension one. Let $O(\mathscr{F})$ denote the family of open, $\mathscr{F}$-saturated subsets of M. Let $U \in O(\mathscr{F})$, and let L be a leaf of $\mathscr{F}|U$. Smoothness of class C^2 implies that there exists a compact, transverse one-manifold $R \subset U$ such that every leaf of $\bar{L} \cap U$ meets $\mathrm{int}(R)$ [C-C 1, (3.7)]. Consequently, $\bar{L} \cap U$ contains a minimal set of $\mathscr{F}|U$ [C-C 1, (3.0)].

DEFINITION. An $\mathscr{F}$-saturated subset $X \subseteq M$ is a local minimal set (LMS) of $\mathscr{F}$ if there exists $U \in O(\mathscr{F})$ such that X is a minimal set of $\mathscr{F}|U$.

Every proper leaf is a LMS, with $U = M \setminus (\bar{L} \setminus L)$. If $U \in O(\mathscr{F})$ and each leaf of $\mathscr{F}|U$ is dense in U, then U itself is a LMS. Finally, an exceptional LMS is one of neither of these types. If X is exceptional, then the transverse manifold $R \subset U$ can be chosen so that $C = X \cap R$ is a Cantor set and misses ∂R.

These LMS play a key role in the structure theory of compact, C^2-foliated manifolds of codimension one [C-C 1]. Our very incomplete understanding of the exceptional type constitutes a major gap in the theory.

Let X be an exceptional LMS, with U, R, and C as above. The holonomy of $\mathscr{F}|U$ induces a C^2 pseudogroup Γ on R for which C is a Γ-minimal set. Let $\Gamma|C$ denote the induced pseudogroup on C. It frequently happens that the choice of R can be made so that $\Gamma|C$ is generated by the local restrictions of a single transformation $\tau: C \rightarrow C$ which, in a sense to be made precise in §1, is essentially a one-sided subshift of finite type (also known as a topological Markov chain [Wa, p. 119]).

DEFINITION. If there exists $\tau: C \rightarrow C$ as above, then C is a Markov

[1] Partially supported by N.S.F. Contract DMS-8420322
[2] Partially supported by N.S.F. Contract DMS-8420956