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THE HELICOIDAL SURFACES AS BONNET SURFACES

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1. Introduction. In this paper we deal with the following question: which surfaces in the Euclidean space E^3 admit a mean-curvature-preserving isometry which is not an isometry of the whole space? This question has been studied by a series of mathematicians beginning with Bonnet [1] and including Cartan [2] and Chern [3]. These surfaces are special and have been classified into three types:

1. The surfaces of constant mean curvature other than the plane and the sphere;
2. Certain surfaces of nonconstant mean curvature that admit a one-parameter family of geometrically distinct nontrivial isometries;
3. Certain surfaces of nonconstant mean curvature that admit a single nontrivial isometry unique up to an isometry of the whole space.

A surface that belongs to one of the above types is called a *Bonnet surface*.

By a nontrivial isometry of a surface we mean an isometry of the surface to another surface or to itself which does not extend to an isometry of the whole space. Two isometries are said to be geometrically distinct if one is not the composition of the other followed by a space-isometry.

A *helicoidal surface* in E^3 is the locus of an appropriately chosen curve under a *helicoidal motion* with pitch in the interval $(0, \infty)$. Such a motion is described by a one-parameter group of isometries in E^3. The orbits of this motion (helices) through the initial curve foliate the helicoidal surface. More details can be found in various places in the literature—for instance in [4].

The main result of this work states:

THEOREM. *The helicoidal surfaces are necessarily Bonnet surfaces and they represent all three types.*

REMARKS. 1. The helicoidal surfaces of the third type provide a negative answer to the following conjecture, posed by Lawson and Tribuzy [5]:

We consider a Riemannian surface Σ and a smooth function $H: \Sigma \to \boldsymbol{R}$.