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THE EXPONENT OF CONVERGENCE OF POINCARÉ SERIES ASSOCIATED WITH SOME DISCONTINUOUS GROUPS

Dedicated to Professor Tadashi Kuroda on his sixtieth birthday

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1. Introduction. Let $\boldsymbol{R}^{n+1}$ be the $(n+1)$-dimensional Euclidean space $(n \geqq 1)$. Each point of $\boldsymbol{R}^{n+1}$ is denoted by a column vector $v = {}^t(v_1, v_2, \cdots, v_{n+1})$, where t denotes the transpose. We put $|v| = \{\sum_{i=1}^{n+1} (v_i)^2\}^{1/2}$ and $x_{n+1}(v) = v_{n+1}$. Let $\boldsymbol{B}^{n+1} = \{v \in \boldsymbol{R}^{n+1}: |v| < 1\}$ and $\boldsymbol{H}^{n+1} = \{v \in \boldsymbol{R}^{n+1}: x_{n+1}(v) > 0\}$ be the open unit ball and the upper half space in $\boldsymbol{R}^{n+1}$, respectively. We denote by $S(x)$ the n-sphere in $\boldsymbol{R}^{n+1}$ with center at x and radius 1.

A Möbius transformation of $\boldsymbol{R}^{n+1} \cup \{\infty\}$ is, by definition, a composite of a finite number of inversions in $\boldsymbol{R}^{n+1} \cup \{\infty\}$ with respect to n-spheres or n-planes. Let Möb be the group of all the Möbius transformations of $\boldsymbol{R}^{n+1} \cup \{\infty\}$. We denote by $|\gamma'(x)|$ the $(n+1)$-th root of the absolute value of the determinant of the Jacobian matrix of $\gamma \in$ Möb at $x \in \boldsymbol{R}^{n+1} \setminus \{\gamma^{-1}(\infty)\}$.

An element $\gamma \in$ Möb with a fixed point at ∞ is of the form $\gamma(x) = \lambda Ax + v$ for some $\lambda > 0$, $A \in O(n+1)$ and $v \in \boldsymbol{R}^{n+1}$, where $O(n+1)$ is the group of orthogonal matrices of degree $n+1$ (see [1, p. 20]). Next assume that $\gamma(\infty) \neq \infty$. Then, for the inversion σ with respect to $S(\gamma^{-1}(\infty))$, we have $\gamma \circ \sigma(\infty) = \infty$ so that $\gamma \circ \sigma(x) = \lambda Ax + v$. Hence $\gamma(x) = \lambda A\sigma(x) + v$. Therefore $|\gamma'(x)| = \lambda/|x - \gamma^{-1}(\infty)|^2$ since $|\sigma'(x)| = 1/|x - \gamma^{-1}(\infty)|^2$. Let the center and the radius of the n-sphere $\{x \in \boldsymbol{R}^{n+1}: |\gamma'(x)| = 1\}$ be $\alpha(\gamma)$ and $\rho(\gamma)$, respectively. Then we have $\alpha(\gamma) = \gamma^{-1}(\infty)$ and $\rho(\gamma)^2 = \lambda$ so that

$$|\gamma'(x)| = \rho(\gamma)^2/|x - \alpha(\gamma)|^2 \,. \tag{1}$$

Further, let the interior and the exterior of the n-sphere be $I(\gamma)$ and $E(\gamma)$, respectively. Then, as in [1, p. 30],

$$\gamma(E(\gamma)) = I(\gamma^{-1}) \,, \quad \gamma(I(\gamma)) = E(\gamma^{-1}) \,. \tag{2}$$

Let Möb$(\boldsymbol{B}^{n+1})$ be the subgroup of Möb whose elements map $\boldsymbol{B}^{n+1}$ onto itself. A subgroup Γ of Möb$(\boldsymbol{B}^{n+1})$ is said to be discontinuous if the orbit $\{\gamma(o)\}_{\gamma \in \Gamma}$ of the origin $o \in \boldsymbol{B}^{n+1}$ under Γ has no accumulation points in $\boldsymbol{B}^{n+1}$. Hence, for a discontinuous subgroup Γ, the set $\Lambda(\Gamma)$ of accumulation points of $\{\gamma(o)\}_{\gamma \in \Gamma}$ is contained in $\partial \boldsymbol{B}^{n+1}$. We call $\Lambda(\Gamma)$ the limit set of Γ. Let $\delta(\Gamma)$