Tôhoku Math. J.
40 (1988), 651-671.

SPHERICAL FUNCTIONS OF HERMITIAN AND SYMMETRIC FORMS III

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(Received July 1, 1987)

Introduction. In the previous papers [2] and [3], we have introduced and studied spherical functions and a spherical transform on the space of nondegenerate hermitian, or symmetric, matrices over a $\mathfrak{p}$-adic number field. In [2], we have shown the injectivity of the spherical transform, and in [3] we have closely studied the case of matrices of size 2. In this paper, making use of the results in [3], we shall show the functional equations for spherical functions and determine their possible poles.

Let k be a $\mathfrak{P}$-adic number field with $\mathfrak{P}$ not lying over 2, $\mathcal{O}$ the ring of integers of k and Π a prime element of k. Let X be the space of nondegenerate symmetric matrices of size n with entries in k. Then $K = GL_n(\mathcal{O})$ acts on X by $k\cdot x = kx^tk$, $k \in K$, $x \in X$. For $x \in X$, a character $\chi = (\chi_1, \cdots, \chi_n)$ of $(k^*/k^{*2})^n$ and $s = (s_1, \cdots, s_n) \in \boldsymbol{C}^n$, consider the following integral:

$$(*) \qquad L(x; \chi; s) = \int_{K'} \prod_{i=1}^{n} |d_i(k\cdot x)|^{s_i} \chi_i(d_i(k\cdot x))dk ,$$

where dk is the Haar measure on K normalized by $\int_K dk = 1$, $d_i(k\cdot x)$ is the determinant of the upper left i by i block of $k\cdot x$, and $K' = \{k \in K: \prod_{i=1}^{n} d_i(k\cdot x) \neq 0\}$.

The right hand side of $(*)$ is absolutely convergent for $\mathrm{Re}(s_1), \cdots, \mathrm{Re}(s_{n-1}) \geqq 0$, and has an analytic continuation to a rational function in $q^{s_1}, \cdots, q^{s_n}$ (cf. [1]). Thus we may regard $L(x; \chi; s)$ as an element in $C^\infty(K\backslash X)$, the space of all K-invariant complex-valued functions on X. We call $L(x; \chi; s)$ a spherical function on X.

We introduce a new variable $z = (z_1, \cdots, z_n)$ which is related to s as follows:

$$\begin{cases} s_i = -z_i + z_{i+1} - \dfrac{1}{2} & (1 \leqq i \leqq n-1) \\ s_n = -z_n + \dfrac{n-1}{4} . \end{cases}$$

Partly supported by the Grants-in-Aid for Encouragement of Young Scientists, The Ministry of Education, Science and Culture, Japan.