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UNIFORM STABILITY FOR ONE-DIMENSIONAL DELAY-DIFFERENTIAL EQUATIONS WITH DOMINANT DELAYED TERM

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1. Introduction. For $q \geqq 0$ and $\alpha, \beta \in \boldsymbol{R}$, the one-dimensional differential-difference equation

$$x'(t) = -\alpha x(t) - \beta x(t-q) \tag{1.1}$$

is a simple example of a delay-differential equation and has been studied from early times in the development of the stability theory of delay-differential equations. For (1.1), the theory of characteristic equations is valid and it is known that the zero solution of (1.1) is uniformly stable if and only if α and β satisfy one of the following conditions:

(R_1) $\quad \alpha \geqq |\beta|$,

(R_2) $\quad \alpha = \beta \sin \eta, \quad 0 \leqq \beta q \leqq \left(\eta + \dfrac{\pi}{2}\right) \Big/ \cos \eta, \quad -\dfrac{\pi}{2} < \eta < \dfrac{\pi}{2}$,

(R_3) $\quad -\alpha = \beta, \quad 0 \leqq \beta q < 1$,

that is, (α, β) is contained in the region (stability region) illustrated in Figure 1 with its boundary except for the point $(-1/q, 1/q)$. Moreover, the zero solution of (1.1) is uniformly asymptotically stable if and only if (α, β) is contained in the interior of $R_1 \cup R_2$ (cf. [3], [7]). It is a feature that R_2 and R_3 become smaller as q increases, while R_1 is independent of q.

On the other hand, the theory of characteristic equations is not applicable to the delay-differential equation such as

$$x'(t) = -a(t)x(t) - b(t)x(t-r(t)), \tag{1.2}$$

where $a, b: [0, \infty) \to \boldsymbol{R}$ and $r: [0, \infty) \to [0, q]$ are continuous functions. Liapunov's method seems to be the only way to investigate the behavior of solutions of (1.2). For (1.2), it is reasonable to expect a similar stability region for (α, β) under the conditions

$$0 \leqq \alpha \leqq a(t), \qquad |b(t)| \leqq \beta \tag{1.3}$$

or

$$\alpha \leqq a(t) \leqq 0, \qquad 0 \leqq b(t) \leqq \beta. \tag{1.4}$$