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# CLASS NUMBERS OF POSITIVE DEFINITE BINARY AND TERNARY UNIMODULAR HERMITIAN FORMS

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CONTENTS



## 0. Introduction.

0.1. This paper is a continuation of [32] . Let $(V, H)$ be a positive definite Hermitian space over an imaginary quadratic field $K$, and let $\mathscr{L}$ be a genus of $\mathcal{O}$-lattices in $V$ with respect to the unitary group $\mathbb{G}:=\mathbb{U}(V, H)$, where $\mathcal{O}$ is the ring of integers of $K$. As we saw in [32], the class number $\boldsymbol{h}(\mathscr{L})$ of $\mathscr{L}$ is expressed as a finite sum:

$$\boldsymbol{h}(\mathscr{L})=\sum_{f\in F}\sum_{[g]_{\boldsymbol{Q}}}\boldsymbol{h}([g]_{\boldsymbol{Q}};\mathscr{L}),\tag{0.1}$$

where in the first sum $f$ runs through the set of characteristic polynomials of the torsion elements of $\mathbb{G}$; and the second sum is taken over the *locally integral* $G$-conjugacy classes $[g]_{\boldsymbol{Q}}=[g]_G$ which belong to $f$, and the invariants $\boldsymbol{h}([g]_{\boldsymbol{Q}};\mathscr{L})$ are given by

$$\boldsymbol{h}([g)_{\boldsymbol{Q}};\mathscr{L})=\sum_{\mathscr{L}(V)}\mathbb{M}(V)\prod_{p}c_p(g, U_p, V_p),\tag{0.2}$$

with $\mathbb{M}(V)$ the mass of an idèlic arithmetic subgroup $V$ of the centralizer $G(g)_{\mathbb{A}}$ of $g$ in $G_{\mathbb{A}}$. See [32] for a more precise definition. We note among others that the masses were evaluated there.

In the present paper, we shall carry out the computations of the local factors $c_p(g, U_p, V_p)$, and derive from (0.1), (0.2) explicit formulas for the class numbers of genera consisting of *unimodular* Hermitian lattices, of ranks **two** and **three**.

0.2. To state our main results, let $K=\boldsymbol{Q}(\sqrt{-m})$ ($m>0$, square free) be an imaginary quadratic field, and let $\mathcal{O}$ be its ring of integers. Let $V$ be a vector space of dimension $n$ over $K$, which is equipped with a positive definite Hermitian form $H$: