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CRITERIA FOR QUASI-SYMMETRICITY AND THE HOLOMORPHIC SECTIONAL CURVATURE OF A HOMOGENEOUS BOUNDED DOMAIN

Dedicated to Professor Ichiro Satake on his sixtieth birthday

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Introduction. It is well-known that to every biholomorphic equivalence class of homogeneous bounded domains in $\boldsymbol{C}^n$, there naturally corresponds bijectively an isomorphism class of normal j-algebras of dimension $2n$. On the base of normal j-algebras, curvature properties of the Bergman metric on a homogeneous bounded domain were discussed in [4], [5], [6], [7], [8], [9].

In this paper we first define the notion of a "strong j-ideal" of a normal j-algebra such that the decomposition of a normal j-algebra into simple strong j-ideals is precisely related to the decomposition of the corresponding homogeneous bounded domain into irreducible ones as Riemannian manifolds with respect to the Bergman metrics (Lemma 1.4).

Let $(\mathfrak{g}, j)$ be a normal j-algebra with $\mathfrak{n}=[\mathfrak{g}, \mathfrak{g}]$ and $\mathfrak{a}$ be the orthogonal complement of $\mathfrak{n}$. Let

$$\mathfrak{g}=\sum \mathfrak{n}_{ab}+\sum j\mathfrak{n}_{ab}+\sum \mathfrak{n}_{a\infty}$$

be the root space decomposition of $\mathfrak{g}$ with respect to the adjoint representation of $\mathfrak{a}$ on $\mathfrak{n}$. Let $\mathscr{L}=\sum \mathfrak{n}_{ab}$, $\mathscr{U}=\sum \mathfrak{n}_{a\infty}$. For $x\in\mathscr{L}$, we define two endomorphisms of $\mathscr{L}$ by $A(x)=2^{-1}((\mathrm{ad}_{\mathscr{L}}jx)+(\mathrm{ad}_{\mathscr{L}}jx)^t)$, $D(x)=2^{-1}((\mathrm{ad}_{\mathscr{L}}jx)-(\mathrm{ad}_{\mathscr{L}}jx)^t)$, and an endomorphism of $\mathscr{U}$ by $\varphi(x)=(\mathrm{ad}_{\mathscr{U}}jx)+(\mathrm{ad}_{\mathscr{U}}jx)^t$. For $x, y\in\mathscr{L}$, let $x\cdot y=A(x)y$. Then, $(\mathscr{L}, \cdot)$ is a commutative distributive algebra over $\boldsymbol{R}$. If $(\mathscr{L}, \cdot)$ is a Jordan algebra, the corresponding homogeneous bounded domain is said to be quasi-symmetric in the sense of Satake [16] (cf. [11]). We consider the following conditions on $(\mathfrak{g}, j)$:

(J) $(\mathscr{L}, \cdot)$ is a Jordan algebra.

(D) For every $x\in\mathscr{L}$, $D(x)$ is a derivation of $(\mathscr{L}, \cdot)$.

(M) For each simple strong j-ideal $\tilde{\mathfrak{g}}$ of $(\mathfrak{g}, j)$, all root spaces $\mathfrak{n}_{ab}\subset\tilde{\mathfrak{g}}$ $(a<b)$ are of the same dimension and so are all $\mathfrak{n}_{a\infty}\subset\tilde{\mathfrak{g}}$.

(A) $2\varphi(x\cdot y)=\varphi(x)\circ\varphi(y)+\varphi(y)\circ\varphi(x)$ on $\mathscr{U}$ for all $x, y\in\mathscr{L}$.

The main purpose of the present paper is to show that (J), (D), and (M) are mutually equivalent, that (M) implies (A), and that if $(\mathfrak{g}, j)$ is simple with $\mathscr{U}\neq\{0\}$, then (A) implies