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WEAK AND CLASSICAL SOLUTIONS OF THE TWO-DIMENSIONAL MAGNETOHYDRODYNAMIC EQUATIONS

Dedicated to Professor Shōzō Koshi on his sixtieth birthday

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Introduction. Let Ω be a bounded domain in $\boldsymbol{R}^2$ with smooth boundary $\partial\Omega$. In $Q_T := \Omega\times(0, T)$, we consider the following magnetohydrodynamic equations for an *ideal* incompressible fluid coupled with magnetic field:

$$(*)\quad \begin{aligned} &\partial_t u+(u, \nabla)u-(B, \nabla)B+\nabla((1/2)|B|^2)+\nabla\pi=f && \text{in } Q_T\,, \\ &\partial_t B-\Delta B+(u, \nabla)B-(B, \nabla)u=0 && \text{in } Q_T\,, \\ &\operatorname{div} u=0\,, \qquad \operatorname{div} B=0 && \text{in } Q_T\,, \\ &u\cdot\nu=0\,, \qquad B\cdot\nu=0 \qquad \operatorname{rot} B=0 && \text{on } \partial\Omega\times(0, T)\,, \\ &u\big|_{t=0}=u_0\,, \qquad B\big|_{t=0}=B_0\,. \end{aligned}$$

Here $u=u(x, t)=(u^1(x, t), u^2(x, t))$, $B=B(x, t)=(B^1(x, t), B^2(x, t))$ and $\pi=\pi(x, t)$ denote the unknown velocity field of the fluid, magnetic field and pressure of the fluid, respectively; $f=f(x, t)=(f^1(x, t), f^2(x, t))$ denotes the given external force, $u_0=u_0(x)=(u_0^1(x), u_0^2(x))$ and $B_0=B_0(x)=(B_0^1(x), B_0^2(x))$ denote the given initial data and ν denotes the unit outward normal on $\partial\Omega$.

The first purpose of this paper is to show the existence and uniqueness of a *global weak solution* of $(*)$ without restriction on the data. In case B is identically equal to zero, i.e., in the case of the Euler equations, such a problem for *global weak* and *classical solutions* was solved by Bardos [1] and Kato [8], respectively. (Kikuchi [9] extended the result of Kato [8] in an exterior domain.) Using the energy method developed by Bardos [1], we can obtain a *global weak solution* in our case.

Our second purpose is to show the existence and uniqueness of a *local classical solution* of $(*)$. Although the method of characteristic curves for the vorticity equation plays an important role in a *global classical solution* of the two-dimensional Euler equations, such a method seems to give us only a *local classical solution* of $(*)$ because of the additional terms $(B, \nabla)B$ and $(u, \nabla)B-(B, \nabla)u$. Our result on classical solutions, however, can be regarded as a generalization of that of Kato [8] in some sense.

We shall devoted Section 1 to preliminaries and definition of a weak solution of