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A THEOREM ON THE LIMIT SETS OF QUASICONFORMAL DEFORMATIONS OF INFINITELY GENERATED FUCHSIAN GROUPS OF THE FIRST KIND

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1. Introduction. Let Γ be a Fuchsian group. Denote by $\Lambda(\Gamma)$ and $\Omega(\Gamma)$ its limit set and region of discontinuity, respectively. Then Γ is said to be of the first kind if $\Omega(\Gamma)$ is not connected. If all elements of $\Gamma\setminus\{1\}$ are hyperbolic transformations, Γ is said to be purely hyperbolic. Let w be a quasiconformal automorphism of the Riemann sphere $\hat{C}$ which is compatible with Γ, that is, $w\circ\gamma\circ w^{-1}$ is a Möbius transformation for each $\gamma\in\Gamma$. Then $w\Gamma w^{-1}$ is a Kleinian group and is called a quasiconformal deformation of Γ. The limit set $\Lambda(w\Gamma w^{-1})$ coincides with $w(\Lambda(\Gamma))$, which is a quasicircle when Γ is of the first kind. For two Jordan curves J_1 and J_2 in the finite complex plane C we define the Fréchet distance $[J_1, J_2]$ as $\inf\max\{|z_1(t)-z_2(t)|;\, 0\leqq t\leqq 1\}$, where the infimum is taken over all possible parametrizations $z_k(t)$ of J_k $(k=1, 2)$.

In Chu [1] the following theorem is used as a key lemma to prove a theorem on the outradii of the Teichmüller spaces of finitely generated purely hyperbolic Fuchsian groups of the first kind.

THEOREM A. *Let J be a rectifiable Jordan curve in C and let $\delta>0$. Then there exists a quasiconformal deformation G of a finitely generated purely hyperbolic Fuchsian group of the first kind so that* $[\Lambda(G), J]<\delta$.

Theorem A is proved by means of a theorem of Maskit on finitely generated Kleinian groups (Maskit [4, Theorem 2]). The assumption of the rectifiability of J can be removed (see Lemma 4.1). In this note we prove the following theorem, which is an analogue of Theorem A.

THEOREM B. *Let J be a Jordan curve in C and let $\delta>0$. Then there exists a quasiconformal deformation G of an infinitely generated Fuchsian group of the first kind so that* $[\Lambda(G), J]<\delta$.

We prove Theorem B by constructing a group G explicitly. In §2 we prove two lemmas which are used in §4. In §3 we construct a quasiconformal mapping used in §5. In §4 we construct an infinitely generated Kleinian group G whose limit set $\Lambda(G)$ is

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