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RESILIENT LEAVES IN TRANSVERSELY AFFINE FOLIATIONS

Dedicated to Professor Akio Hattori on his sixtieth birthday

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1. Introduction. Let $\mathscr{F}$ be a codimension one foliation on a manifold M. We say that $\mathscr{F}$ is *transversely affine* if M is covered by a collection of $\mathscr{F}$-distinguished charts for which the coordinate transformations are affine (*i.e.*, of the form $x \mapsto ax+b$, $a \neq 0$) in the direction transverse to $\mathscr{F}$. See [6, I, Chap. III] for fundamental properties of codimension one transversely affine foliations.

The purpose of this paper is to study the problem of existence of resilient leaves in codimension one transversely affine foliations. Here, a leaf is said to be *resilient* if it is nonproper and with nontrivial holonomy. Resilient leaves are classified into two types—locally dense type and exceptional type. It is known that locally dense resilient leaves appear in some codimension one transversely affine foliations on closed manifolds (See, *e.g.*, [1] or Step 2 of §2 in this paper).

In [5], Furness and Fedida asserted that a codimension one transversely affine foliation cannot have exceptional leaves. But their proof seems to have a gap. In fact, it is rather easy to give a counterexample on an *open* manifold.

Now the first result of this paper is stated as follows:

THEOREM 1.1. *There exists a codimension one transversely affine foliation on a closed* 3*-manifold which contains an exceptional minimal set.*

By a classical theorem of Sacksteder [10], this exceptional minimal set contains a resilient leaf, necessarily of exceptional type.

REMARK. After circulating the earlier draft of this paper, the author received a letter from G. Hector to the effect that he constructed a similar example several years ago and will write his result up in the near future.

Let Aff($\boldsymbol{R}$) be the group of affine transformations of the real line. A codimension one transversely affine foliation $\mathscr{F}$ on a manifold M induces a holonomy homomorphism $h\colon \pi_1(M) \to \mathrm{Aff}(\boldsymbol{R})$. We call the image of h the *global holonomy group* of $\mathscr{F}$. The next result characterizes the existence of locally dense resilient leaves in terms of the global holonomy group.

THEOREM 1.2. *Let $\mathscr{F}$ be a codimension one transversely oriented, transversely affine foliation on a closed manifold and Γ its global holonomy group. Then either of the following*