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CERTAIN ASPECTS OF TWISTED LINEAR ACTIONS, II

Dedicated to Professor Akio Hattori on his 60th birthday

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0. Introduction. In the previous papers [1], [2], we have introduced the concept of a twisted linear action which is an analytic action of a non-compact Lie group on a sphere.

We have shown that there are uncountably many topologically distinct analytic actions of $\boldsymbol{SL}(n, \boldsymbol{R})$ on an $(nk-1)$-sphere for each $n>k\geqq 2$. Furthermore, we have shown that there are uncountably many C^1-differentiably distinct but topologically equivalent analytic actions of $\boldsymbol{SL}(n, \boldsymbol{R})$ on a k-sphere for each $k\geqq n\geqq 2$.

In this paper, we shall show other aspects of twisted linear actions. In particular, we shall show that there are uncountably many C^2-differentiably distinct but C^1-differentiably equivalent analytic actions of $\boldsymbol{R}^n$ on an n-sphere for each n.

1. Twisted linear actions. Here we recall the definition of twisted linear actions. Throughout this paper, a matrix means only the one with real coefficients.

1.1. Let $\boldsymbol{u}=(u_i)$ and $\boldsymbol{v}=(v_i)$ be column vectors in $\boldsymbol{R}^n$. As usual, we define their inner product by $\boldsymbol{u}\cdot\boldsymbol{v}=\sum_i u_i v_i$ and the length of $\boldsymbol{u}$ by $\|\boldsymbol{u}\|=\sqrt{\boldsymbol{u}\cdot\boldsymbol{u}}$. Let $M=(m_{ij})$ be a square matrix of degree n. We say that M satisfies the condition (T) if the quadratic form

$$\boldsymbol{x}\cdot M\boldsymbol{x}=\sum_{i,j} m_{ij}x_i x_j$$

is positive definite. It is easy to see that M satisfies (T) if and only if

$$(\mathrm{T}') \qquad \frac{d}{dt}\|\exp(tM)\boldsymbol{x}\|>0 \qquad \text{for each} \quad \boldsymbol{x}\in\boldsymbol{R}_0^n=\boldsymbol{R}^n-\{\boldsymbol{0}\}\,, \quad t\in\boldsymbol{R}\,.$$

If M satisfies (T′), then

$$\lim_{t\to+\infty}\|\exp(tM)\boldsymbol{x}\|=+\infty \quad \text{and} \quad \lim_{t\to-\infty}\|\exp(tM)\boldsymbol{x}\|=0$$

for each $\boldsymbol{x}\in\boldsymbol{R}_0^n$, and hence there exists a unique real valued analytic function τ on $\boldsymbol{R}_0^n$

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