Tôhoku Math. J.
42 (1990), 17–44

CONSTRUCTION AND STABILITY ANALYSIS OF TRANSITION LAYER SOLUTIONS IN REACTION-DIFFUSION SYSTEMS

KUNIMOCHI SAKAMOTO

(Received August 8, 1988, revised July 24, 1989)

1. Introduction. In mathematical biology, reaction-diffusion equations have been of great interest as a model describing spatial pattern formations. One of the most powerful approaches to the existence of spatially inhomogeneous solutions is a singular perturbation method. In fact, this method enables us to construct solutions with sharp spatial transition layers [5], [6], [12], [13]. It is the purpose of this paper to present a method to construct solutions with internal transition layers in the context of singular perturbation problems. We also emphasize the stability analysis of the solutions so obtained as above. Our method is slightly different from those in [5], [6], [12], [13] in that existence and stability analysis are carried out simultaneously.

For d_i, $i=1, 2$, positive parameters, consider the following pair of reaction-diffusion equations

$$\text{(PDE)} \qquad \begin{cases} u_t = d_1 u_{xx} + f(u, v) \\ v_t = d_2 v_{xx} + g(u, v) \end{cases} \qquad x \in (0, 1), \quad t > 0$$

under the homogeneous Neumann boundary conditions

$$\text{(BC)} \qquad u_x = 0 = v_x \qquad x = 0, 1, \quad t > 0 .$$

The problem (PDE)+(BC) has been studied rather extensively for the case in which both diffusion coefficients d_1, d_2 are very large by, among others, Conway, Hoff and Smoller [3], Hale [9] and Hale and Rocha [10]. Roughly speaking, the asymptotic dynamics of (PDE)+(BC) is qualitatively the same as that of

$$\text{(ODE)} \qquad u_t = f(u, v) , \qquad v_t = g(u, v)$$

when min (d_1, d_2) is sufficiently large.

On the other hand, there has been a series of works by Nishiura, Mimura, et al. [6], [7], [8], [12], [13], [14], [15] on (PDE)+(BC) from a viewpoint of pattern formation when $d_1 > 0$ is very small with d_2 remaining large. These authors have established the existence of equilibrium solutions with interior transition layers [13] as well as their

This research was supported in part by National Science Foundation of the United States under NSF grant DMS-8507056.