Tôhoku Math. J.
42 (1990), 1–16

THE EXISTENCE OF LIMIT CYCLES OF NONLINEAR OSCILLATION EQUATIONS*

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(Received August 6, 1988, revised August 1, 1989)

1. Introduction. Many mathematicians have done a lot of work on the existence of limit cycles of the Liénard equation

$$x''+f(x)x'+g(x)=0\,, \tag{1}$$

and many good results have been obtained (see [5]–[7]). It is worthwhile to generalize these results to more general nonlinear equation. Huang [3] [4] considered the existence of limit cycles of the equation

$$x''+f(x)\eta(x')x'+\psi(x')g(x)=0\,. \tag{2}$$

In this paper we use a new method to deal with the existence of limit cycles of the equation (2) and obtain some new results. Our results generalize the well-known theorems of Dragilev [1] and Filippov [2].

2. A system equivalent to (2). Our basic idea in this paper is to find a closed orbit of a two-dimensional system equivalent to the given equation. There are several ways of obtaining such an equivalent system. It is well-known that each of the systems

$$\begin{cases} x'=y \\ y'=-f(x)y-g(x) \end{cases} \tag{3}$$

and

$$\begin{cases} x'=y-F(x) \\ y'=-g(x) \end{cases}\,, \tag{4}$$

where $F(x)=\int_0^x f(s)ds$, is considered to be a system equivalent to the Liénard equation (1). For the general equation (2), it is quite easy to consider an equivalent system

$$\begin{cases} x'=y \\ y'=-f(x)\eta(y)y-\psi(y)g(x) \end{cases} \tag{5}$$

corresponding to (3), but until now, we have never seen any method generalizing the equivalent system (4) for the more complicated equation (2). In this paper, we offer

* Supported by the National Natural Science Foundation of China.