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DIRICHLET SERIES CORRESPONDING TO SIEGEL'S MODULAR FORMS OF DEGREE n WITH LEVEL N

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0. Introduction.

0.1. Koecher [KO] introduced Dirichlet series corresponding to Siegel's modular forms and presented explicit formulas for the Dirichlet series which express the location of the poles and the residues in a satisfactory manner, but did not succeed in proving them. Maass in his lecture notes [MA, § 15] studied those Dirichlet series in full generality and obtained the analytic continuation and the functional equations for them. His method is based upon the theory of invariant differential operators acting on real symmetric matrices, which gives a powerful tool in investigating those Dirichlet series and their functional equations. However one cannot have precise information on the residues of the poles by his method. In [AR] we have proved Koecher's explicit formulas by using Klingen's Eisenstein series and the structure theorem for the space of Siegel's modular forms due to Klingen [KL1]. Recently Weissauer [WE] studied Koecher-Maass Dirichlet series corresponding to Siegel's cusp forms with level N and solved a certain converse problem concerning the correspondence between those Dirichlet series with grössen characters and Siegel's cusp forms.

Our aim of the present paper is to prove Koecher's explicit formulas for the Dirichlet series corresponding to Siegel's modular forms (not necessarily cusp forms) with level N without using Klingen's Eisenstein series (Klingen in [KL2, p. 235] suggested the problem of obtaining Koecher's formulas without the help of Klingen's Eisenstein series). We also obtain an explicit formula for the Epstein-Koecher zeta function.

Another more arithmetic aspect of Koecher-Maass Dirichlet series is discussed inBöcherer [BÖ].

0.2. We summarize our results.

Let $\mathfrak{H}_n$ be the Siegel upper half plane of degree n, on which the Siegel modular group $\Gamma^{(n)}=Sp(n, \boldsymbol{Z})$ of degree n acts in a usual manner. Let $\Gamma_0^{(n)}(N)$ be the congruence subgroup of $\Gamma^{(n)}$ with level N given by

$$(0.1)\qquad \Gamma_0^{(n)}(N)=\left\{M=\begin{pmatrix} A & B \\ C & D \end{pmatrix}\in\Gamma^{(n)} \,\middle|\, C\equiv 0 \bmod N\right\}.$$

For a Dirichlet character $\varepsilon \bmod N$ and a positive integer k, denote by $M_k(\Gamma_0^{(n)}(N), \varepsilon)$ the space of all holomorphic functions f on $\mathfrak{H}_n$ satisfying