Tôhoku Math. J.
42 (1990), 195–203

ASYMMETRY OF MAXIMAL FUNCTIONS ON THE AFFINE GROUP OF THE LINE*

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(Received February 20, 1989)

1. Introduction. In this paper, we study the weak-type (1, 1), and strong type (p, p) $(1<p<+\infty)$ boundedness of certain Hardy-Littlewood maximal function operators. These are generated by taking the maximal averages over the left- (resp. right-) translates of various families of neighbourhoods of the identity in the affine, or '$ax+b$', group of the line. Investigations of this kind are basic to the study of singular integral operators. Thus, the aim of the paper is to delineate some of the positive and negative results for maximal functions on this group.

Let H be the group of orientation-preserving affine transformations of the real line commonly known as the '$ax+b$' group. We consider H as the upper half-plane

$$H=\{(u, v): u\in \boldsymbol{R}, v>0\}.$$

The product of two elements is given by

$$(u, v)(s, t)=(u+vs, vt).$$

The left-invariant Haar measure element is $dudv/v^2$ and the modular function $\Delta(u, v)$ is v^{-1}, the convention being that

$$\Delta(h^{-1})\int_H f(g)dg=\int_H f(gh)dg.$$

The group H is isomorphic to the AN part of the standard Iwasawa decomposition of $SL_2(\boldsymbol{R})$. It is a connected, simply connected solvable group.

Another perspective on H is that it can be considered as the symmetric space $SL_2(\boldsymbol{R})/SO(2)$. As such, it has a natural geometric structure: $SL_2(\boldsymbol{R})$ acts on it by left translation; there is a canonical Riemannian metric derived from the Killing form on the Cartan component $\mathfrak{p}$ consisting of the set of 2×2 symmetric matrices of trace 0. In our realisation, the Riemannian metric is $ds^2=(du^2+dv^2)/v^2$, and if $r>0$, the Riemannian ball centred at $e=(0, 1)$ of radius r is the Euclidean disc centred at $(0, \cosh r)$ and of radius $\sinh r$.

1980 *Mathematics Subject Classification* (1985 revision). Primary 43A80; secondary 42B25.

Key words and phrases. Maximal functions, solvable Lie groups.

* Research supported by the Australian Research Council, the Flinders University Research Budget and the Italian Consiglio Nazionale delle Ricerche.