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POINCARÉ SERIES FOR DISCRETE MOEBIUS GROUPS ACTING ON THE UPPER HALF SPACE

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Abstract. Consider the Poincaré series of order t for a discrete Moebius group acting on the n-dimensional upper half-space. If the point at infinity is a horocyclic limit point or a Garnett point, then the series diverges for any positive number t. If the point at infinity is an ordinary point or a cusped parabolic fixed point, then the series converges for any t which is greater than $n-1$. If the point at infinity is an atom for the Patterson-Sullivan measure, then the series converges for any t which is equal to or greater than the critical exponent of the group.

1. Discrete Moebius groups. Let R^n and $\overline{R^n}$ be the n-dimensional Euclidean space and its one-point compactification, respectively. We use the notation $x=(x_1, \dots, x_n) \in R^n$ and when matrices act on x, we treat x as a column vector. The subspace $H^n=\{x \in R^n \mid x_n>0\}$ of R^n is a model for the hyperbolic n-space and supports a metric ρ derived from the differential $d\rho=|dx|/dx_n$. We call H^n the n-dimensional upper half-space.

The (full) Moebius group $M(\overline{R^n})$ is the group of Moebius transformations of $\overline{R^n}$, which is generated by inversions in spheres and reflections in planes. Moebius transformations are classified into three conjugacy classes in $M(\overline{R^n})$ as follows. An element in $M(\overline{R^n})$ is said to be loxodromic if it is conjugate to a transformation of the form

$$\gamma(x)=\lambda Tx\,, \tag{1.1}$$

where $\lambda>0$, $\lambda \neq 1$, and $T \in O(n)$, the group of $n \times n$-orthogonal matrices, and parabolic if it is conjugate to a transformation of the form

$$\gamma(x)=Tx+a\,, \tag{1.2}$$

where $T \in O(n)$, $a \in R^n$ and $Ta=a \neq 0$. A non-trivial element is said to be elliptic if it is neither loxodromic nor parabolic.

By $\gamma'(x)$ we denote the Jacobian matrix of $\gamma \in M(\overline{R^n})$ at $x \in R^n$. For $\gamma \in M(\overline{R^n})$ the chain rule implies that $\gamma'(x)$ can be written as $\gamma'(x)=\nu T(x)$ with $\nu>0$ and $T \in O(n)$. We denote by $|\gamma'(x)|$ this positive number ν and call it the linear magnification of γ at x. For $\gamma \in M(\overline{R^n})$ with $\gamma(\infty) \neq \infty$ the set $I(\gamma)=\{x \in R^n \mid |\gamma'(x)|=1\}$ is an $(n-1)$-sphere with center $\gamma^{-1}(\infty)$. The sphere $I(\gamma)$ is called the isometric sphere of γ. The action of γ on

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