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VECTOR FIELDS AND DIFFERENTIAL FORMS ON GENERALIZED RAYNAUD SURFACES

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Abstract. We consider the tangent and cotangent sheaves of generalized Raynaud surfaces, which have cuspidal fibrations and which are mostly of general type. In particular, we compute the dimensions of the space of vector fields and of non-closed differential 1-forms.

Let $f\colon V\to B$ be a fibration from a smooth projective surface to a smooth projective curve over an algebraically closed field k. In the case of characteristic zero, almost all fibres of f are nonsingular. In the case of positive characteristic, however, there exists fibrations whose general fibres have singularities. Generalized Raynaud surfaces are typical examples of surfaces which have such fibrations. Moreover, these surfaces have interesting geometry in positive characteristic. In the present article, we compute the dimension of the space of vector fields and give an estimate for the difference of the dimension of the space of all global differential 1-forms from that of the space of closed differential 1-forms on generalized Raynaud surfaces.

1. Generalized Raynaud surfaces. Throughout this article, we assume that k is an algebraically closed field of characteristic $p\geqq 3$. To begin with, we define generalized Tango curves. Let C be a smooth projective curve over k and let $\mathcal{N}$ be an invertible sheaf on C with positive degree. Suppose that there exist local sections $\{\xi_i\in\Gamma(U_i,\mathcal{O}_C)\}_{i\in I}$ whose differentials $\{d\xi_i\}$ are local generators of the sheaf of differentials Ω_C^1 satisfying $d\xi_i=a_{ij}^{pn}d\xi_j$, where $\{a_{ij}\}_{i,j\in I}$ are transition functions of $\mathcal{N}$ for an affine open covering $\{U_i\}_{i\in I}$ and where n is a positive integer with $n\not\equiv 0 \pmod p$ and $n>1$. Then we call the triple $(C,\mathcal{N},\{d\xi_i\})$ a *generalized Tango curve of index n*. Note that $\mathcal{N}^{np}\cong\Omega_C^1$. The following lemma is useful for constructing generalized Tango curves. For the proof, we refer to Takeda [7].

LEMMA 1.1 (Kurke [1]). *Let ω be an exact differential on a smooth projective curve C. Suppose that the divisor of ω has the form pnD, where D is a nonzero effective divisor and n is a positive integer with $n\not\equiv 0$ $(\mathrm{mod}\, p)$ and $n>1$. Then there exist local sections*

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