

EXISTENCE AND UNIQUENESS OF SOLUTIONS FOR THIRD ORDER NONLINEAR BOUNDARY VALUE PROBLEMS

ZHAO WEILI

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Abstract. In this paper, we study the existence and uniqueness of the solutions of the general boundary value problems for the third order nonlinear ordinary differential equations. Our results improve some of the known results; moreover, these are very convenient for applications.

1. Introduction. Since 1970, Jackson and several other authors [1]–[12] have made a substantial study for the existence and uniqueness of the solutions for the two-point boundary value problems for third order nonlinear ordinary differential equations.

We discuss in the present paper the existence and uniqueness of the solutions of some general two-point boundary value problems for third order nonlinear ordinary differential equations by making use of third order differential inequalities and by the method of constructing upper and lower solutions.

We consider the third order nonlinear differential equation

$$(1) \quad y''' = f(x, y, y', y'')$$

together with the boundary conditions

$$(2) \quad ay'(0) - by''(0) = A, \quad y(1) = B, \quad y'(1) = C$$

or

$$(3) \quad y(0) = A, \quad y'(0) = B, \quad ay'(1) + by''(1) = C,$$

where A, B, C are constants, a, b are nonnegative constants, and $a + b > 0$.

In Section 2, we state some preliminaries needed in the sequel.

We investigate, in Section 3, the existence and uniqueness of the solutions for the third order nonlinear boundary value problems (1), (2) and (1), (3).

Finally, we give in Section 4 some examples to illustrate the applications of the main results of this paper.

As far as the author knows, the technique of constructing upper and lower solutions