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THE STANDARD CR STRUCTURE ON THE UNIT TANGENT BUNDLE

Dedicated to Professor Takesi Kotake on his sixtieth birthday

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Abstract. The unit tangent bundle of a Riemannian manifold is one of popular examples of contact manifolds. It has the standard CR structure which is not integrable in general. We study the recently defined gauge invariant of type (1,3) of the CR structure and show that the invariant vanishes, if and only if the Riemannian manifold is of constant curvature -1.

Introduction. Popular examples of contact manifolds are the odd-dimensional spheres and the unit tangent bundles of Riemannian manifolds. These examples have the standard contact Riemannian structures and their associated CR structures.

A contact Riemannian structure satisfying the integrability condition $Q=0$ corresponds to a strongly pseudo-convex integrable CR structure. There are rich results in the study of strongly pseudo-convex integrable CR structures. If one wants to generalize the Chern-Moser-Tanaka invariant of (1,3)-type of CR structures to a (1,3)-type invariant of gauge transformations of contact Riemannian structures, it seems to be necessary that one fixes a linear connection (Tanno [12]) or a nowhere vanishing m-form on the contact manifold M, where $\dim M=m$ (cf. §3).

In §4 we give the expression for our (1,3)-type invariant B of the standard contact Riemannian structure on the unit tangent bundle of a Riemannian manifold.

THEOREM A. *Let* (M, g) *be a Riemannian manifold of dimension* $m\geq 3$ *and* (T^1M, η, g^*) *be its unit tangent bundle with the standard contact Riemannian structure* (η, g^*). *Then the gauge invariant* B *of* (1,3)-*type of* (T^1M, η, g^*) *vanishes, if and only if* (M, g) *is of constant curvature* -1.

It is worth noticing that if (M, g), $m\geq 3$, is of constant curvature -1 then the CR structure associated with the contact Riemannian structure (η, g^*) on the unit tangent bundle T^1M is integrable and yet the natural almost complex structure of the ambient space, i.e., the tangent bundle TM, is not integrable.

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