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CURVATURE PINCHING THEOREM FOR MINIMAL SURFACES WITH CONSTANT KAEHLER ANGLE IN COMPLEX PROJECTIVE SPACES, II

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Abstract. We consider minimal surfaces with constant Kaehler angle in complex projective spaces. By using J-invariant higher order osculating spaces and pinched Gaussian curvature, we give characterization theorems for these minimal surfaces.

This is a continuation of our paper [12]. For each integer p with $0 \leq p \leq n$, it is known that there exists a full isometric minimal immersion $\varphi_{n,p}: S^2(K_{n,p}) \to P^n(C)$ of a 2-dimensional sphere of constant Gaussian curvature $K_{n,p} = 4\rho/(n+2p(n-p))$ into the complex projective n-space with the Fubini-Study metric of constant holomorphic sectional curvature 4ρ (cf. [1] and [2]). In [12], using J-invariant first order osculating spaces, we gave characterization theorems for immersions $\varphi_{n,p}$ for $p \leq 3$. The purpose of this paper is to generalize these to the case of $\varphi_{n,p}$ for $p \geq 4$ (cf. Section 4). To study the problem, we use J-invariant higher order osculating spaces to find some scalars defined globally on M, and calculate their Laplacians (cf. Section 6). In this paper, we use the same terminology and notation as in [12] unless otherwise stated.

4. *J*-invariant higher order osculating spaces and the main theorems. Let X be a Kaehler manifold of complex dimension n of constant holomorphic sectional curvature 4ρ and $x: M \to X$ an isometric immersion of an oriented 2-dimensional Riemannian manifold M into X. Let $C(s)$ be a smooth curve in M through a point $p = C(0)$ of M with parameter s proportional to the arc length. We denote by D^kC/ds^k the k-th covariant derivative along $C(s)$ in X. Let $T_p^{(k)}(C)$ be a subspace of $T_p(X)$ spanned by $\{DC/ds, JDC/ds, \dots, D^kC/ds^k, JD^kC/ds^k\}$ at $s=0$, where J is the complex structure of X. $T_p^{(k)}$ is defined to be the subspace spanned by all $T_p^{(k)}(C)$ for curves C lying on M through p and is called the *J-invariant k-th osculating space* of M at p. We then have $T_p(M) \subset T_p^{(1)} \subset \cdots \subset T_p^{(m)} \subset T_p(X)$. Let $O_p^{(k+1)}$ be the orthogonal complement of $T_p^{(k)}$ in $T_p^{(k+1)}$ and N_p^m the orthogonal complement of $T_p^{(m)}$ in $T_p(X)$, so that we have $T_p^{(k+1)} = T_p^{(k)} + O_p^{(k+1)}$ and $T_p(X) = T_p^{(m)} + N_p^m$. We put $O_p^1 = T_p^{(1)}$. Note that we have

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