Tôhoku Math. J.
47 (1995), 1–13

ON THE KRIEGER-ARAKI-WOODS RATIO SET

GAVIN BROWN, ANTHONY H. DOOLEY AND JANE LAKE

(Received February 24, 1992, revised July 18, 1994)

Abstract. We show how to calculate the ratio sets of G-measures as limit points of infinite products of the associated g-functions. In particular, we show that every g-measure is of type III_1.

1. Introduction. By Dye's celebrated theorem, every ergodic system of type II or type III is orbit equivalent to one of the form (X, Γ, μ), where X is the infinite product of two-point spaces, Γ the (countable) group of finite coordinate changes in X, and μ some measure on X which is quasi-invariant and ergodic with respect to the action of Γ. The Krieger-Araki-Woods ratio set, discussed in [9], is an invariant for orbit equivalence, allowing classification into systems of types II_1, II_∞, III_1, III_λ $(0<\lambda<1)$, and III_0. We will discuss here only probability measures.

In a recent paper [1], two of the authors introduced the G-measure formalism, showing that all ergodic measures may be regarded as a generalization of the g-measures of M. Keane, that is, there are functions g_k on X such that

$$\frac{d\mu}{d\mu^{(n)}}(x)=g_1(x)g_2(x)\cdots g_n(x)=G_n(x)\,.$$

Here, $\mu^{(n)}$ denotes the measure μ averaged over the first n coordinates, and the function g_i depends on the coordinates $(x_i, x_{i+1}, \cdots)$ and satisfies

$$\frac{1}{2}(g_i(0, x_{i+1}, x_{i+2}, \cdots)+g_i(1, x_{i+1}, x_{i+2}, \cdots))=1 \quad \text{for every} \quad x\in X\,.$$

In this paper, we shall seek to characterise the ratio set of μ in terms of the limit points of infinite products of the form $\prod_{i=n}^{\infty} g_i(u)/g_i(v)$, $u, v\in X$. Our major result, Theorem 4.4, gives a necessary and a different sufficient condition which are nevertheless rather close to each other, for a number r to belong to the ratio set. In Section 5, this theorem is applied to show that provided the image of g contains an interval, every g-measure is of type III_1. Hence by a theorem of Connes-Krieger [3], [5], they are all orbit equivalent. In the last section, we apply our results to infinite product measures.

We gratefully acknowledge the support of the Australian Research Council. The second-named author would like to thank the Department of Mathematics, Université

1991 *Mathematics Subject Classification*. Primary 28D99; Secondary 47A35.