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# A REMARK ON THE RIEMANN-ROCH FORMULA ON AFFINE SCHEMES ASSOCIATED WITH NOETHERIAN LOCAL RINGS

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**Abstract.** The aim of this paper is to describe the Riemann-Roch map on affine schemes associated with Noetherian local rings. The Riemann-Roch theorem on singular affine schemes is one of the powerful tools in the commutative ring theory. Our main theorem enables us to calculate the Riemann-Roch maps under some assumption.

**1. Main theorem.** Let $k$ be a field and $R=\bigoplus_{i\geq 0} R_i$ a graded Noetherian ring which satisfies $R_0=k$ and $R=R_0[R_1]$, i.e., $R$ is a graded $k$-algebra generated by $R_1$. We denote by $\mathfrak{m}$ the homogeneous maximal ideal $\bigoplus_{i>0} R_i$.

For an abelian group $M$, we write $M_{\boldsymbol{Q}}$ for $M\otimes_{\boldsymbol{Z}}\boldsymbol{Q}$, where $\boldsymbol{Z}$ (resp. $\boldsymbol{Q}$) is the ring of integers (resp. the field of rational numbers).

Let $X=\mathrm{Proj}(R)$ be a smooth projective variety over $k$ of dimension $d$. We denote by $\mathrm{A}_*(X)=\bigoplus_{i=0}^{d}\mathrm{A}_i(X)$ the *Chow group* of $X$. If we put $\mathrm{CH}^i(X)=\mathrm{A}_{d-i}(X)$ for $i=0,\dots,d$, then $\mathrm{CH}(X)=\bigoplus_{i=0}^{d}\mathrm{CH}^i(X)$ has the structure of a commutative ring (since $X$ is smooth), and is called the *Chow ring* of $X$. (We refer the reader to [5] for definitions and basic facts.)

We put $c=c_1(\mathcal{O}_X(1))\cap[X]\in\mathrm{A}_{d-1}(X)_{\boldsymbol{Q}}=\mathrm{CH}^1(X)_{\boldsymbol{Q}}$, where $c_1(\mathcal{O}_X(1))$ denotes the *first Chern class* of the invertible sheaf $\mathcal{O}_X(1)$, i.e., $c$ stands for the Cartier divisor corresponding to the line bundle $\mathcal{O}_X(1)$. Let

$$\pi:\mathrm{CH}(X)_{\boldsymbol{Q}}\to\mathrm{CH}(X)_{\boldsymbol{Q}}/(c) \tag{1.1}$$

be the natural surjective ring homomorphism ($(c)$ is the principal ideal of $\mathrm{CH}(X)_{\boldsymbol{Q}}$ generated by $c$), and

$$\tau:\mathrm{K}_0(R_{\mathfrak{m}})_{\boldsymbol{Q}}\to\mathrm{A}_*(\mathrm{Spec}\,R_{\mathfrak{m}})_{\boldsymbol{Q}} \tag{1.2}$$

the Riemann-Roch map for the affine scheme $\mathrm{Spec}\,R_{\mathfrak{m}}$ (cf. [5, chap. 18]), where $\mathrm{K}_0(R_{\mathfrak{m}})$ is the *Grothendieck group* of finitely generated $R_{\mathfrak{m}}$-modules and $\mathrm{A}_*(\mathrm{Spec}\,R_{\mathfrak{m}})$ is the Chow group of $\mathrm{Spec}\,R_{\mathfrak{m}}$.

Our main theorem is the following:

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